The Economics of Space 433: Session 1

An Introduction to Dixit-Stiglitz Preferences

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Dixit-Stiglitz Preferences

- In "classic" Intermediate Micro, we were used to dealing with Cobb-Douglas (C-D) utility
  - Consumer's utility function is: $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$
  - x_1 and x_2 are the quantities consumed of each good (e.g. $x_1 = 2$ oranges, $x_2 = 4$ apples)
  - The consumer then proceeds to maximize this utility function subject to budget constraint: $p_1 x_1 + p_2 x_2 = y$

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- It this class, we will focus on another type of utility: Dixit-Stigliz (D-S)
  - Consumer's utility function is: $U(x_1, x_2) = [x_1^{\frac{\sigma-1}{\sigma}} + x_2^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$
  - As in the C-D case, x_1 and x_2 are the quantities consumed of each good
  - Also as in the C-D case, consumer maximizes utility subject to budget constraint: $p_1 x_1 + p_2 x_2 = y$

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Setup

- In this Appendix, we derive facts and properties regarding consumer choice under D-S preferences

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- Here's our roadmap:

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- Here's our roadmap:
  - Derive facts and properties of C-D consumer choice

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Setup

- In this Appendix, we derive facts and properties regarding consumer choice under D-S preferences
- We also want to show that these facts and properties can be derived in the same way as in the C-D case
- Here's our roadmap:
  - Derive facts and properties of C-D consumer choice
  - Use exactly analogous procedure to derive facts and properties of D-S consumer choice

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Before we Start

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- Another change is that we will denote consumer income by w instead of y

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Before we Start

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  - E.g. $c_1 = 3, c_2 = 6$ means I consume 3 oranges and 6 apples
- Another change is that we will denote consumer income by w instead of y
- Hence, consumer maximization problem is:

$$\max_{c_1, c_2} U(c_1, c_2)$$

$$\text{subject to } p_1 c_1 + p_2 c_2 = w$$

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C-D: the Set-up

- The consumer's choice problem with C-D preferences can be written as:

$$\max_{c_1, c_2} c_1^{\alpha} c_2^{1-\alpha}$$

$$\text{subject to } p_1 c_1 + p_2 c_2 = w$$

- Note that from the consumer's perspective, (p_1, p_2, w, α) are all **exogenous** variables/parameters
  - That is, they are just "given" to the consumer. She has no direct influence on their value
  - Consumer takes them as inputs when making her consumption choice
- On the other hand, from the consumer's perspective, c_1, c_2 are **endogenous** variables
  - The consumer chooses their value within her maximization program in order to maximize utility
  - We denote the specific "maximizing" values of c_1, c_2 that the consumer chooses as (c_1^*, c_2^*) and the corresponding "optimized" level of utility as U^*

C-D: Lagrangian and FOCs

- To solve the maximization probem, we start by writing the Lagrangean function

$$\mathcal{L} = c_1^{\alpha} c_2^{1-\alpha} + \mu \left(w - p_1 c_1 - p_2 c_2 \right)$$

- The variable μ is the Lagrange multiplier
- Then take the first-order condition (FOC) of the Lagrangian with respect to c_1 and c_2

$$\frac{\partial \mathcal{L}}{\partial \mathrm{c}_1} = \alpha c_1^{\alpha-1} c_2^{1-\alpha} - \mu p_1 = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial \mathrm{c}_2} = (1-\alpha) c_1^{\alpha} c_2^{-\alpha} - \mu p_2 = 0 \quad (2)$$

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$$\frac{\partial \mathcal{L}}{\partial \mathrm{c}_1} = \alpha c_1^{\alpha-1} c_2^{1-\alpha} - \mu p_1 = 0 \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \mathrm{c}_2} = (1-\alpha) c_1^{\alpha} c_2^{-\alpha} - \mu p_2 = 0 \tag{2}$$

- And don't forget the budget constraint

$$p_1 c_1 + p_2 c_2 = w \tag{3}$$

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C-D: Algebra 1

- Rewrite equations (1) and (2) to isolate the Lagrange multiplier μ in the right-hand side

$$\alpha c_1^{\alpha-1} c_2^{1-\alpha} = \mu p_1$$

$$(1-\alpha) c_1^{\alpha} c_2^{-\alpha} = \mu p_2$$

- Divide the first of these two equations by the second

$$\frac{\alpha c_1^{\alpha-1} c_2^{1-\alpha}}{(1-\alpha) c_1^{\alpha} c_2^{-\alpha}} = \frac{\mu p_1}{\mu p_2} \implies$$

$$\frac{\alpha c_2}{(1-\alpha) c_1} = \frac{p_1}{p_2} \implies$$

$$c_2 = \frac{(1-\alpha) p_1 c_1}{\alpha p_2} \tag{4}$$

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C-D: Algebra 2

- Now substitute c_2 from equation (4) in the budget constraint (3)

$$p_1 c_1 + p_2 [\frac{(1-\alpha) p_1 c_1}{\alpha p_2}] = w \Longrightarrow$$

$$p_1 c_1 [1 + \frac{(1-\alpha)}{\alpha}] = w \Longrightarrow$$

$$c_1^* = \frac{\alpha w}{p_1} \tag{5}$$

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- We have expressed c_1 as a function of exogenous parameters only! Thus, we've found the consumer's utility-maximizing choice, c_1^*
- What about c_2^*? To find it, substitute the c_1 in equation (4) using the optimal c_1^* from equation (5)

$$c_2^* = \frac{(1-\alpha)p_1}{\alpha p_2}[\frac{\alpha w}{p_1}]$$

$$c_2^* = \frac{(1-\alpha)w}{p_2} \tag{6}$$

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C-D: Solution, Utility

- We have found the utility-maximizing consumption bundle! It is the following

$$(c_1^*, c_2^*) = (\frac{\alpha w}{p_1}, \frac{(1-\alpha)w}{p_2})$$

- To find the ``optimized'' utility level achieved by this optimal consumption choice, plug c_1^*, c_2^* into the utility function:

$$U^* = U(c_1^*, c_2^*) = (c_1^*)^\alpha (c_2^*)^{1-\alpha} \Longrightarrow$$

$$U^* = (\frac{\alpha w}{p_1})^\alpha (\frac{(1-\alpha)w}{p_2})^{1-\alpha} \Longrightarrow$$

$$U^* = w(\frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{p_1^\alpha p_2^{1-\alpha}}) \Longrightarrow$$

$$U^* = \frac{w}{\frac{p_1^\alpha p_2^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}} \tag{7}$$

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C-D: Price Index

- From last slide, the "optimized" level of utility achieved by the consumer is

$$U^* = \frac{w}{\frac{p_1^\alpha p_2^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}}$$

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- If we defined a new variable P as $P = \frac{p_1^\alpha p_2^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$, we can then rewrite the utility level simply as

$$U^* = \frac{w}{P}$$

- This saves a lot of notation!

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$$U^* = \frac{w}{P}$$

- This saves a lot of notation!
- P is known as the **price index**
  - Note that P is increasing in p_1 and p_2
  - Also note that the formula for P resembles that of C-D utility
    - Thus, the precise form that the index assumes is influenced by the underlying utility function
    - If we had used a different utility function, the index may have been different!

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C-D: Expenditure Shares

- Finally, let's compute the **expenditure shares** of each good
  - That is, the fraction of the consumer's income that is spent on each good
  - E.g. expenditure share of good 1 is: $\lambda_1 = \frac{\text{money spent on good 1}}{\text{total income}}$

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- So expenditure share of good i is $\lambda_i = \frac{p_i c_i^*}{w}$
- Substituting the values for from equations (5) and (6), we get

$$\lambda_1 = \alpha$$

$$\lambda_2 = 1 - \alpha$$

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- Substituting the values for from equations (5) and (6), we get

$$\lambda_1 = \alpha$$

$$\lambda_2 = 1 - \alpha$$

- Note that expenditure share is independent of prices or income
  - E.g. if p_1 goes up, consumer buys fewer units of c_1 but spends more money on each unit
  - These two effects cancel out so that expenditure share on good 1 stay constant at α

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Moving on to D-S

- Having shown the facts and properties of consumer choice under Cobb-Douglas preferences, we now move on to Dixit-Stiglitz preferences
- We will use the exact same sequence of steps as the ones we used to derive results for Cobb-Douglas preferences
- To make it even more explicit, we will present derivations for C-D and D-S side-by-side

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C-D vs. D-S: Set-Up

- The consumer's utlity-maximization problem can be written in each case (C-D and D-S) as follows

Cobb-Douglas case

$$\max_{c_1, c_2} c_1^{\alpha} c_2^{1-\alpha} \quad \text{subject to:} \quad p_1 c_1 + p_2 c_2 = w \tag{8}$$

Dixit-Stiglitz case

$$\max_{c_1, c_2} [c_1^{\frac{\sigma-1}{\sigma}} + c_2^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \quad \text{subject to:} \quad p_1 c_1 + p_2 c_2 = w \tag{9}$$

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C-D vs. D-S: Lagrangean and FOCs

- Now, let us write the Lagrangean and take first-order conditions:

Cobb-Douglas case

$$\mathcal{L} = c_1^{\alpha} c_2^{1-\alpha} + \mu(w - p_1 c_1 - p_2 c_2) \quad (10)$$

$$\text{FOC}(c_1): \frac{\partial \mathcal{L}}{\partial c_1} = \alpha c_1^{\alpha-1} c_2^{1-\alpha} - \mu p_1 = 0 \quad (12)$$

$$\text{FOC}(c_2): \frac{\partial \mathcal{L}}{\partial c_2} = (1-\alpha) c_1^{\alpha} c_2^{-\alpha} - \mu p_2 = 0 \quad (13)$$

Dixit-Stiglitz case

$$\mathcal{L} = [c_1^{\frac{\sigma-1}{\sigma}} + c_2^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} + \mu(w - p_1 c_1 - p_2 c_2) \quad (11)$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = (\frac{\sigma}{\sigma-1})[c_1^{\frac{\sigma-1}{\sigma}} + c_2^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1}(\frac{\sigma-1}{\sigma})c_1^{\frac{\sigma-1}{\sigma}-1} - \mu p_1 = 0 \quad (14)$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = (\frac{\sigma}{\sigma-1})[c_1^{\frac{\sigma-1}{\sigma}} + c_2^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1}(\frac{\sigma-1}{\sigma})c_2^{\frac{\sigma-1}{\sigma}-1} - \mu p_2 = 0 \quad (15)$$

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C-D vs. D-S: Algebra 1

- Algebraically manipulating equations (12)-(15) a bit, we get the following:

Cobb-Douglas case

$$\alpha c_1^{\alpha-1} c_2^{1-\alpha} = \mu p_1$$

$$(1-\alpha) c_1^{\alpha} c_2^{-\alpha} = \mu p_2$$

Dixit-Stiglitz case

$$[c_1^{\frac{\sigma-1}{\sigma}} + c_2^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1} c_1^{\frac{\sigma-1}{\sigma}-1} = \mu p_1$$

$$[c_1^{\frac{\sigma-1}{\sigma}} + c_2^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1} c_2^{\frac{\sigma-1}{\sigma}-1} = \mu p_2$$

- Dividing the top equation by the bottom equation, we get:

$$\frac{\alpha c_2}{(1-\alpha)c_1} = \frac{p_1}{p_2} \Rightarrow \qquad c_2 = \frac{(1-\alpha)p_1}{\alpha p_2} c_1 \qquad (16)$$

$$\frac{c_1^{\frac{\sigma-1}{\sigma}-1}}{c_2^{\frac{\sigma-1}{\sigma}-1}} = \frac{p_1}{p_2} \Rightarrow \qquad c_2 = \left(\frac{p_2}{p_1}\right)^{-\sigma} c_1 \qquad (17)$$

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C-D vs. D-S: Algebra 2

- Substituting equations (16)-(17) into budget constraints, we get:

Cobb-Douglas case

$$p_1 c_1 + p_2 \left(\frac{(1-\alpha) p_1}{\alpha p_2} c_1 \right) = w \Rightarrow$$

$$c_1^* = \frac{\alpha w}{p_1} \quad (18)$$

Dixit-Stiglitz case

$$p_1 c_1 + p_2 \left((\frac{p_2}{p_1})^{-\sigma} c_1 \right) = w \Rightarrow$$

$$c_1^* = \frac{p_1^{-\sigma} w}{p_1^{1-\sigma} + p_2^{1-\sigma}} \quad (19)$$

- We just obtained c_1^*, the optimal consumption choice for good 1!
- Now plug it back into equations (16)-(17) to obtain the optimal consumption choice for good 2:

$$c_2^* = \frac{(1-\alpha) w}{p_2} \quad (20)$$

$$c_2^* = \frac{p_2^{-\sigma} w}{p_1^{1-\sigma} + p_2^{1-\sigma}} \quad (21)$$

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C-D vs. D-S: Solution, Utility

- Thus, the utility-maximizing choice bundle in each case is:

Cobb-Douglas case

$$(c_1^*, c_2^*) = \left(\frac{\alpha w}{p_1}, \frac{(1-\alpha)w}{p_2}\right)$$

Dixit-Stiglitz case

$$(c_1^*, c_2^*) = \left(\frac{p_1^{-\sigma} w}{p_1^{1-\sigma} + p_2^{1-\sigma}}, \frac{p_2^{-\sigma} w}{p_1^{1-\sigma} + p_2^{1-\sigma}}\right)$$

- We can then find the achieved level of utility $U^* = U(c_1^*, c_2^*)$ by plugging c_1^* and c_2^* into the utility function:

$$\begin{aligned} U^* &= (c_1^*)^{\alpha}(c_2^*)^{1-\alpha} \\ &= \frac{w}{\left(\frac{p_1^{\alpha} p_2^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)} \end{aligned} \qquad (22)$$

$$\begin{aligned} U^* &= [(c_1^*)^{\frac{\sigma-1}{\sigma}} + (c_2^*)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\left(\frac{p_1^{-\sigma} w}{p_1^{1-\sigma} + p_2^{1-\sigma}}\right)^{\frac{\sigma-1}{\sigma}} + \left(\frac{p_2^{-\sigma} w}{p_1^{1-\sigma} + p_2^{1-\sigma}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ &= \frac{w}{[p_1^{1-\sigma} + p_2^{1-\sigma}]^{\frac{1}{1-\sigma}}} \end{aligned} \qquad (23)$$

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C-D vs. D-S: Price Index

- For each case, let us conveniently define the price index, P, so that we can express utility as $U^* = \frac{w}{P}$:

Cobb-Douglas case

$$P \equiv \frac{p_1^{\alpha} p_2^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \qquad (24)$$

Dixit-Stiglitz case

$$P \equiv [p_1^{1-\sigma} + p_2^{1-\sigma}]^{\frac{1}{1-\sigma}} \qquad (25)$$

- Note that, in both cases, increases in either p_1 or p_2 cause the price index to increase
- Also note that, in both cases, the formula of the price index bear some similarities to the utility function
  - Once more, the form of the index is influenced by the underlying utility function

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C-D vs. D-S: Expenditure Shares

- As we mentioned above, the expenditure share of a good i is $\lambda_i = \frac{p_i c_i^*}{w}$, for $i \in \{1, 2\}$
- So, substituting the values of c_i^* from equations (18)-(21), we get in each case:

Cobb-Douglas case

$$\begin{aligned} \lambda_1 &= \alpha \\ \lambda_2 &= 1 - \alpha \end{aligned} \tag{26}$$

Dixit-Stiglitz case

$$\begin{aligned} \lambda_1 &= \frac{p_1^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}} \\ \lambda_2 &= \frac{p_2^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}} \end{aligned} \tag{27}$$

- Expenditure shares are affected by prices in the D-S case, but not in the C-D case
  - In the D-S case, a higher p_1 makes the consumer spend less money on good 1
  - That is, the ''fewer units'' effect dominates the ''higher price per unit'' effect
- In both the C-D and D-S case, $\lambda_1 + \lambda_2 = 1$
  - This makes sense: if I add up the fractions of my income that I spend on each good, I must get 100%

D-S: Expenditure Shares

- Note the structure of the formula for D-S expenditure shares:

$$\lambda_1 = \frac{p_1^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}}$$

$$\lambda_2 = \frac{p_2^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}}$$

- For each good, the formula has the structure $\frac{K_{\text{"that good"}}}{K_{\text{"summed across all goods"}}}$
- It turns out that this structure generalizes to the D-S case with not only 2 but N goods (which we didn't tackle in this Appendix)
  - In that case, the expenditure share of each good i would be: $\lambda_i = \frac{p_i^{1-\sigma}}{\sum_{s=1}^{N} p_s^{1-\sigma}}$ for $i \in \{1, 2, ..., N\}$

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