The Economics of Space 433: Lectures 19 and 20

Gains from Reducing Spatial Frictions

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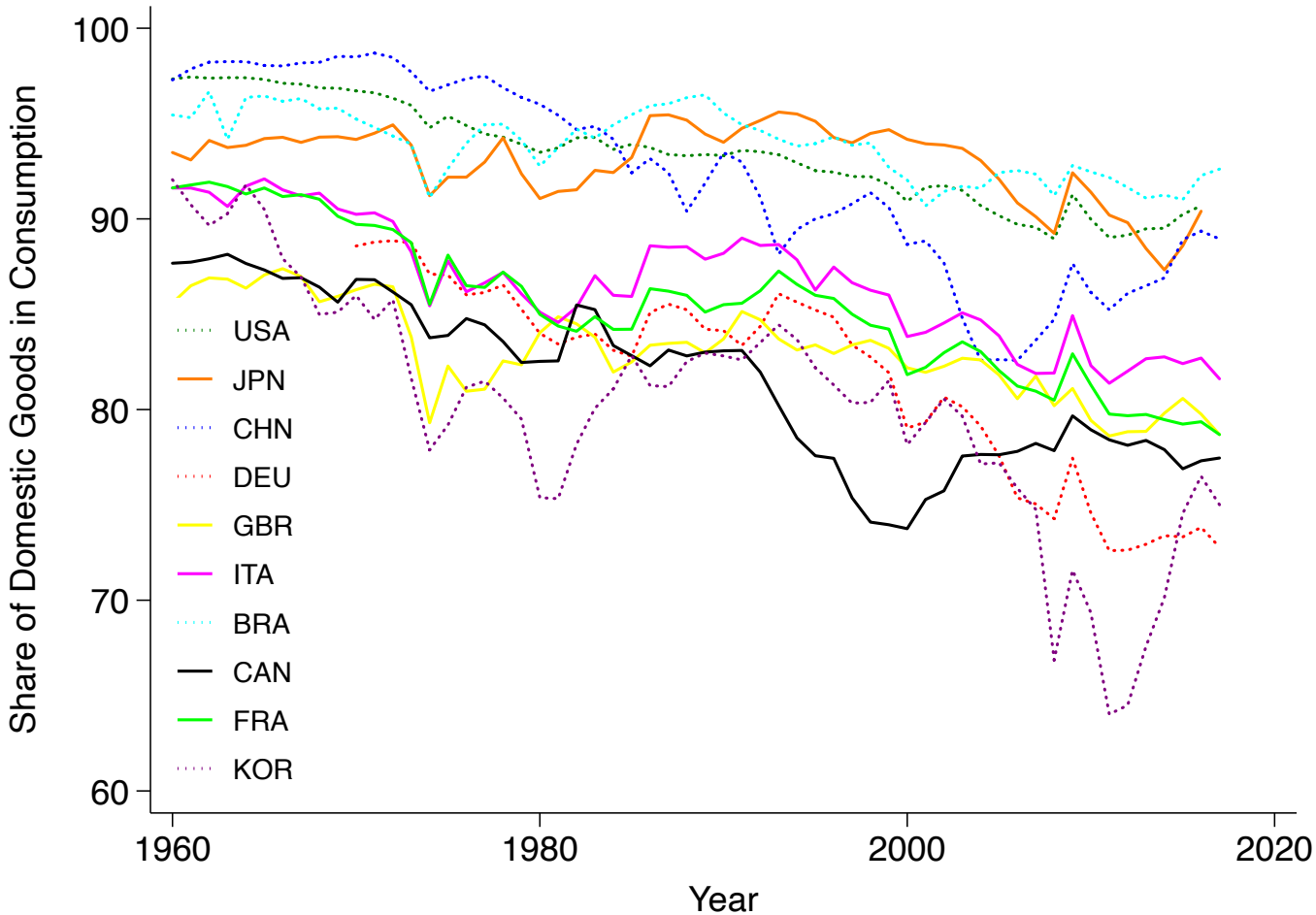
International Spatial Economics

- The purpose of this lectures is to broaden the perspective of space to the world level
- We want to consider countries as separated locations
- Consider the movements of firms and goods across countries, but not of people (migration at the intra country level more common)
- We are motivated by the gains from international integration and ask what is the impact of the integrated world economy?

Roadmap

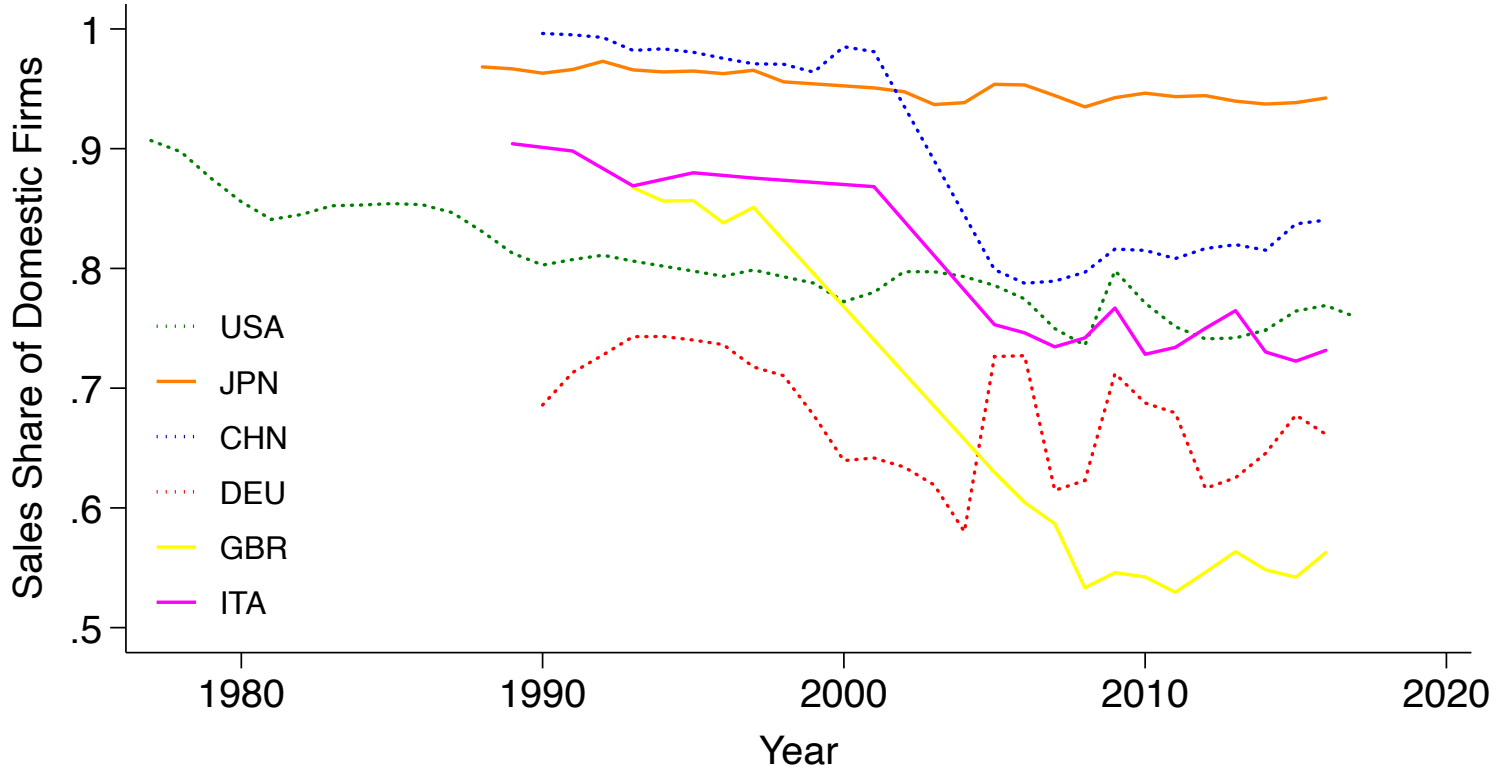
- **Globalization and Openness**
- A Simple Model to Compute the Gains from Trade
- Globalization andMultinational Activity
- A Simple Model to Compute the Gains from Multinationals

Globalization: Share of Spending on Domestic Goods



Using data from the World Input-Output Tables and the World Bank

Globalization: Share of Production Done by Domestic Firms



Note: Data for the US is from the BEA. Data up to 2005 for non US countries is from UNCTAD. Data after 2005 for non US countries is from OECD. Note that the definition of foreign affiliate of UNCTAD is an incorporated or unincorporated enterprise in which an investor, who is a resident in another economy, owns a stake of 10% or more of the ordinary shares or voting power for an incorporated enterprise, or its equivalent for an unincorporated enterprise. The OECD definition of foreign affiliate is firms with at least 50% foreign ownership. The BEA definition of foreign affiliate is an enterprize with 10 percent or more of the voting securities if the enterprise is incorporated or an equivalent interest if the enterprise is unincorporated.

Gains from Openness

- What are the gains from Openness?
  - Potential gains from opening to financial markets (e.g. insurance to aggregate shocks).
    - These gains are the topics of a course in International Finance
  - Potential gains from trade (e.g. increased specialization).
  - Potential gains from foreign investment (e.g. technology transfer).

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A Simple Model to Compute the Gains from Trade

- We will take our regular setup we used in the class
  - As before we assume perfect competition and differentiated varieties
  - But we assume there is no labor mobility
  - We will call this setup, the ‘‘trade model’’
- Assumptions:
  - 2 countries. Country 1 produces good 1, country 2 produces good 2.
  - Representative consumer in each country.

Firms

- Firms produce the good of their country using labor
  - Iceberg trade costs τ_{12} and τ_{21} if the good is exported
  - Domestic prices $p_{11} = w_1/A_1$, $p_{22} = w_2/A_2$ and export prices $p_{12} = w_1\tau_{12}/A_1$, $p_{21} = w_2\tau_{21}/A_2$

Consumers

- Representative consumer with Constant Elasticity of Substitution preferences over the two goods:

$$U(c_{1j}, c_{2j}) = \left((c_{1j})^{\frac{\sigma-1}{\sigma}} + (c_{2j})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

  - c_{1j}, c_{2j} consumption of goods 1,2 and σ elasticity of substitution

- Number of immobile consumers in the two countries: L_1, L_2.
  - Argument generalizes for more than 2 goods/countries
  - Without loss of generality let us focus on $j = 1$
  - Notice: no mobility, so no need to think about amenities

Market Shares

- We can compute the trade shares, i.e., the share of spending on goods from a given country. The domestic share of spending is

$$\lambda \equiv \lambda_{11} \equiv \frac{p_{11} c_{11}}{p_{11} c_{11} + p_{21} c_{21}}$$

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$$\lambda \equiv \lambda_{11} \equiv \frac{p_{11} c_{11}}{p_{11} c_{11} + p_{21} c_{21}}$$

- Recall that the solution of consumption is $c_{11} = (p_{11}/p_{21})^{-\sigma} c_{21}$. Thus:

$$\lambda_{11} = \frac{p_{11} \left(\frac{p_{11}}{p_{21}}\right)^{-\sigma} c_{21}}{p_{11} \left(\frac{p_{11}}{p_{21}}\right)^{-\sigma} c_{21} + p_{21} c_{21}} = \frac{p_{11}^{1-\sigma}}{p_{11}^{1-\sigma} + p_{21}^{1-\sigma}} = \frac{p_{11}^{1-\sigma}}{P_1^{1-\sigma}}$$

where $P_1^{1-\sigma} \equiv p_{11}^{1-\sigma} + p_{21}^{1-\sigma}$ is the CES price index, a weighted mean over prices.

Welfare

- Welfare is the real income in this simple setup, i.e., wage divided by the price index

$$W_1 = w_1/P_1$$

- Recall $p_{11} = \frac{w_1}{A_1}$ and thus

$$\lambda \equiv \lambda_{11} = \frac{p_{11}^{1-\sigma}}{P_1^{1-\sigma}} \implies \lambda = (A_1)^{\sigma-1}\left(\frac{w_1}{P_1}\right)^{1-\sigma} \implies$$

$$\frac{w_1}{P_1} = A_1\lambda^{1/(1-\sigma)}$$

- We will denoted henceforth the domestic share of spending as λ
  - Ignore A_1 (we set to 1) as it does not play a crucial role in this argument
  - We will reinstate its role in the next exercise

Sufficient Statistics for Gains from Trade

- All the above generealize with more locations (Arkolakis, Costinot, Rodriguez-Clare '12)
  - Welfare is a function of the domestic share of spending λ and the elasticity of demand $1-\sigma$!
  - A generalization of a result of Eaton Kortum '02 for a wide class of spatial models

- In changes (denoted with $\hat{}$),

$$\hat{W} = \left(\frac{\hat{w}_1}{\hat{P}_1}\right) = \hat{\lambda}^{1/(1-\sigma)}$$

- To compute gains from trade, we simply need to know $\hat{\lambda}$ and have an estimate for the trade elasticity $\epsilon = 1-\sigma$

Sufficient Statistics for Gains from Trade

- Let us compute the gains from trade
- Import penetration ratio in the USA in 2005 is 9.8% $\implies \lambda = 0.902$
- The elasticity of trade is between -10 and -5 (Anderson, Van Wincoop '03)
- Apply the formula: gains from autarky (where $\lambda = 1$) to trade,

$$\hat{W} = \frac{(\lambda_{trade})^{1/(1-\sigma)}}{(\lambda_{autarky})^{1/(1-\sigma)}} = \left(\frac{.902}{1}\right)^{1/(1-\sigma)} = 1.0103 - 1.021$$

  $\Rightarrow$ The gain ranges from 1.03% to 2.1%.

- With more sectors, gains can increase (e.g. Costinot, Rodriguez-Clare '13)
  - What is then the cost of tarrifs? They lower λ_{trade}
  - Trump tariffs in 2018 only cost 0.04% of GDP (Fajgelbaum et al '20)

The Impact of a Technology or Trade Shock

- Now we turn to understanding the impact of technology and trade shocks to welfare
- Not anymore amenable to previous "sufficient" statistics approach
  - We need to measure the impact of the shock through the trade network (i.e., how τ_{ij} changes are reflected on λ)
- The theoretical underpinnings for the rest of the analysis can be found in Allen, Arkolakis, Takahashi '20 and Adao, Arkolakis, Esposito '21
  - Difference with our previous spatial model: no labor mobility (trade model)
  - Other elements (e.g. costs of trade and product differentiation) remain the same

Motivation: The Impact of a Technology or Trade Shock

- Key question: how to measure impact of a foreign productivity shock
  - We want the aggregate outcome for each region when regions are interconnected
- Autor, Dorn, Hanson '13 estimate the impact of Chinese productivity growth across US regions
  - Key finding: regions exposed to industries China grew the most had relatively adverse outcomes

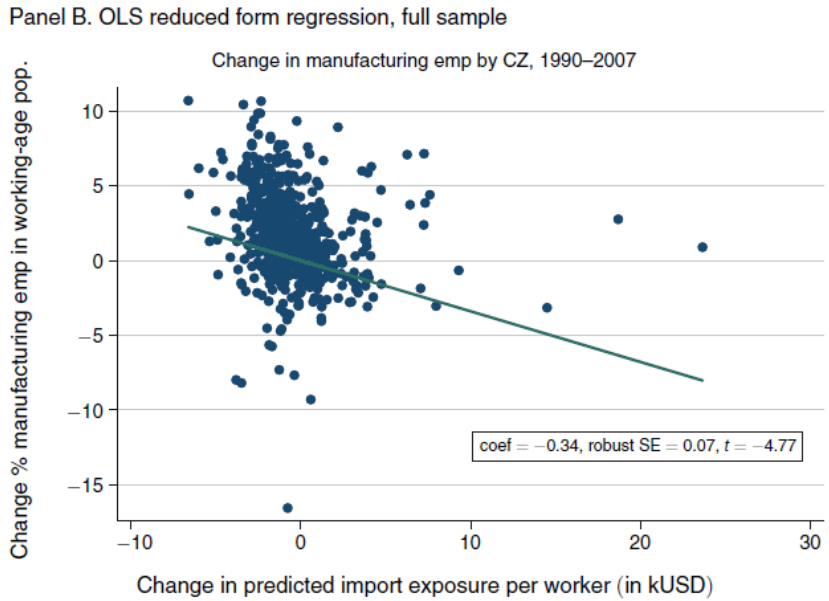

FIGURE 2. CHANGE IN IMPORT EXPOSURE PER WORKER AND DECLINE OF MANUFACTURING EMPLOYMENT: ADDED VARIABLE PLOTS OF FIRST STAGE AND REDUCED FORM ESTIMATES

Notes: $N = 722$. The added variable plots control for the start of period share of employment in manufacturing industries. Regression models are weighted by start of period CZ share of national population.

Multi-Country Trade Equilibrium

- Equilibrium in trade model is very closely related to the previous spatial setup
  - Let us bring productivities back

- Recall from Problem Set 1 spending shares are given by

$$\lambda_{jk} = \frac{p_{jk}^{1-\sigma}}{\sum_{j'} p_{j'k}^{1-\sigma}} = \frac{\left(\frac{w_j}{A_j}\tau_{jk}\right)^{1-\sigma}}{\sum_{j'}\left(\frac{w_{j'}}{A_{j'}}\tau_{j'k}\right)^{1-\sigma}} = \frac{\left(\frac{w_j}{A_j}\tau_{jk}\right)^{1-\sigma}}{P_k^{1-\sigma}}$$

- We also have, like before, an income/trade balance equation

$$w_j L_j = \sum_k \lambda_{jk} w_k L_k$$

Multi-Country Trade Equilibrium

- Income balance equation is:

$$w_j L_j = \sum_k \underbrace{\frac{\left(\frac{w_j}{A_j}\tau_{jk}\right)^{1-\sigma}}{P_k^{1-\sigma}}}_{\text{trade share}} w_k L_k \iff w_j^{\sigma} L_j = A_j^{\sigma-1} \sum_k \frac{(\tau_{jk})^{1-\sigma}}{P_k^{1-\sigma}} L_k w_k \tag{1}$$

  - And price index:

$$P_j^{1-\sigma} = \sum_k \left(\frac{\tau_{kj}}{A_k}\right)^{1-\sigma} w_k^{1-\sigma} \tag{2}$$

Multi-Country Trade Equilibrium

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  - And price index:

$$P_j^{1-\sigma} = \sum_k \left(\frac{\tau_{kj}}{A_k}\right)^{1-\sigma} w_k^{1-\sigma} \tag{2}$$

- Welfare is $W_j = \frac{w_j}{P_j}$ (need not equalize across countries due to labor immobility). Rewrite (1) as

$$W_j^\sigma L_j P_j^\sigma = A_j^{\sigma-1} \sum_k \frac{(\tau_{jk})^{1-\sigma}}{P_k^{1-\sigma}} Y_k \implies W_j^\sigma = \frac{1}{L_j} P_j^{-\sigma} A_j^{\sigma-1} \Pi_j$$

The Effects of A Foreign Productivity Shock

- Totally differentiate assuming $d \log A_j = 0$ for j and all L_k, τ_{jk} for all k, j fixed.

$$W_j^{\sigma} = \frac{1}{L_j} P_j^{-\sigma} A_j^{\sigma-1} \Pi_j \implies d \log W_j = -d \log P_j + \frac{1}{\sigma} d \log \Pi_j$$

i.e., you benefit when price drops but hurt if producer market access declines

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  i.e., you benefit when price drops but hurt if producer market access declines

- Look at the two effects separately. Let y_{jk} be export shares (see Appendix)

$$-d \log P_j = \underbrace{\sum_k \lambda_{kj} d \log A_k}_{\text{efficiency gains}} - \sum_k \lambda_{kj} d \log w_k$$

$$d \log \Pi_j = (\sigma - 1) \underbrace{\sum_k y_{jk} d \log P_k}_{\text{competition effect}} + \sum_k y_{jk} d \log w_k$$

  - Positive foreign shock benefits j through drop in P_j (e.g higher Chinese imports)
  - Hurts j through declines in market access in other regions (P_k elsewhere also drops)

A Smart Guess: Solving for Welfare

- Connection to empirical findings: producer market access may decline creating adverse effects

- What is the net aggregate welfare effect for each region?
  - To compute this, we need to account for changes in other wages etc.
  - Consider special case $\tau_{ij} = \tau_{ji}$. Guess solution: $L_k P_k^{\sigma-1} w_k = A_k^{\sigma-1} w_k^{1-\sigma}$ for all k

- Define $\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1} < 1$ (recall $\sigma > 1 \iff \tilde{\sigma} < 1$, notice $\frac{1-\tilde{\sigma}}{\tilde{\sigma}} = \frac{1-\frac{\sigma-1}{2\sigma-1}}{\frac{\sigma-1}{2\sigma-1}} = \frac{\sigma}{\sigma-1}$)
  - One can show (see Appendix) that

$$(W_j)^{\tilde{\sigma}\sigma} L_j^{\tilde{\sigma}} = A_j^{\tilde{\sigma}(\sigma-1)} \sum_k L_k^{\tilde{\sigma}} (\tau_{jk})^{1-\sigma} A_k^{(\tilde{\sigma}-1)(1-\sigma)} (W_k)^{\tilde{\sigma}(1-\sigma)}$$

- Totally differentiate again with respect to foreign shocks

$$d \log W_j + \frac{\sigma-1}{\sigma} \sum_k y_{jk} d \log W_k = \sum_{k \neq j} y_{jk} d \log A_k$$

Direct and Indirect Effects of the Shock

- Consider a productivity or trade shock, $d \log A_C > 0$, to one country, China,

$$d \log W_j \left(\frac{(\lambda_{jj} + 1)\sigma - \lambda_{jj}}{\sigma} \right) = \underbrace{y_{jC} d \log A_C}_{\text{direct productivity benefit}} \underbrace{- \frac{\sigma - 1}{\sigma} \sum_{k \neq j} y_{jk} d \log W_k}_{\text{export reallocation effect}}$$

  - Direct positive effects from productivity shock
  - But indirectly a cascade of reallocations that could lead to negative effects
    - Negative effects if initial y_{jk} was high in regions with large W_k
    - For example: EU countries benefited a lot from Chinese growth in imports. May lead to substantial reduction in spending to US goods

Comparison of Gains

- The two approaches of computing gains from international markets are giving seemingly different results
  - The ex-post sufficient statistics approach gives always gains
  - The analysis of the "shock" to a foreign market may lead to potential losses

- The reason is that conditional on increases in openness (decreases in λ), gains are always realized in our setup
  - But due to trade reallocation effects, various shocks in the international markets (e.g. growth in country C's productivity) may lead to reduction in trade and increases in λ
  - In this sense, the two approaches are consistent

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Motivation: The Impact of Multinationals

- WSJ (April 19, 2011) on U.S. multinationals during 2000-2010: ‘‘declined employment in the U.S. by 1 million, while increasing employment overseas by 2.4 million”.

How Important are Multinational Companies?

- Production globalization is rapidly taking place across the globe over the past 3 decades.

- Multinational Activity is now prominent in all dimensions of economic life.
  - Leading brands (e.g., Toyota, Volkswagen).
  - Major investors (e.g., Boeing, Airbus).
  - Leading innovators (e.g., Ford spends $8 billion per year for R&D, etc.).

Definitions

- Foreign Direct Investment (FDI): long-term participation of foreign firms in another country.
  - Inward and Outward foreign investment is the received investment from foreign firms to one country or the outward investment from domestic firms to a foreign country.
- Multinational Enterprises (MNE): firms that have headquarters in one country but operate in multiple countries.
  - Inward and Outward Multinational Production.
  - Note: We can observe FDI flows and stocks but also sales of affiliates.
- Multinational Production (MP): production of affiliates of firms in foreign countries

Examples

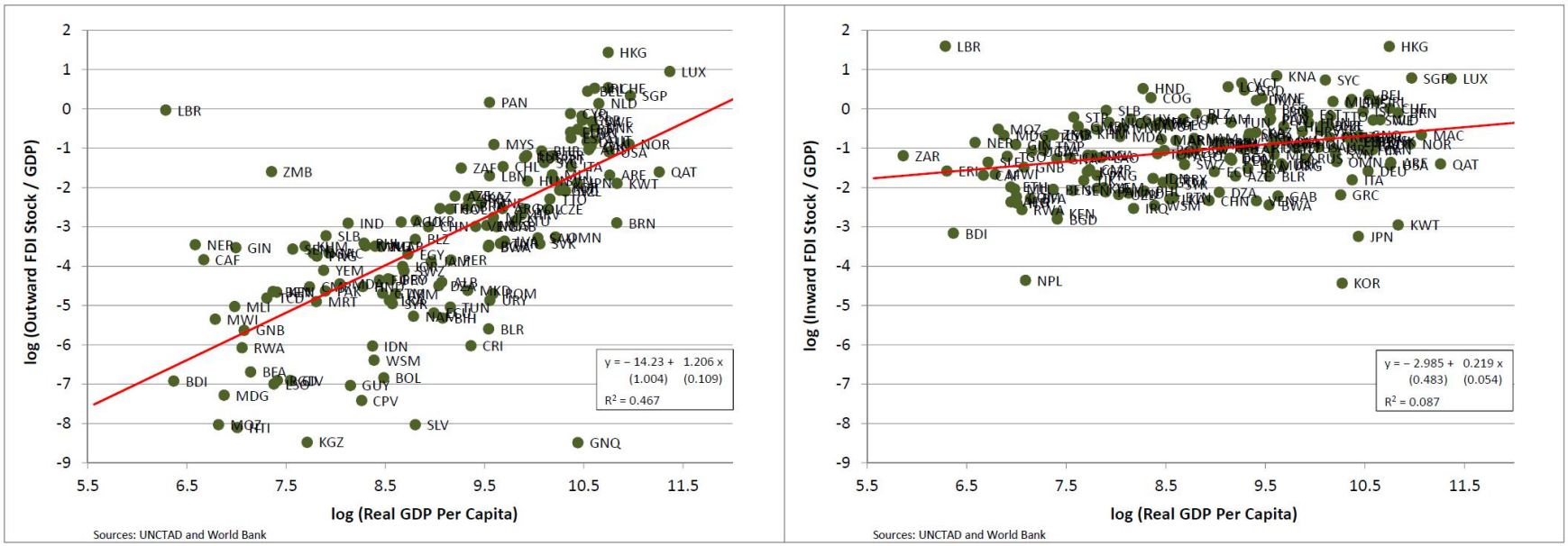
- Examples: Fage Yogurt (best Greek yogurt...outside of Greece!) builds a plant in the US.
  - FDI: $148 million dollar investment for a plant in the USA
  - MP: Fage sells more than $300 mil worth of yogurt in the US
  - MNE activity: Sadly, recently Fage is too much of an MNE... they switched their headquarters to Luxembourg!
  - So it is Greek Yogurt by a Luxembourg firm, produced in the US.

Horizontal and Vertical FDI

- Definition: Horizontal FDI refers to production of similar final goods across the globe.
- Definition: Vertical FDI refers to firms geographically fragmenting their production.
- Examples
  - Fage builds yogurt production plants throughout the world.
  - Fage operates milk production facility in Greece to supply US yogurt production plants.

Stylized Facts: Who Sends and Who Receives

- As Antras and Yeaple '14 point out: Multinational activity is primarily concentrated in developed countries where it is mostly two-way. Developing countries are more likely to be destination of multinational activity than source



FDI inward and outward stock to country GDP. Source: Antras, Yeaple '14

Stylized Facts on Multinationals

- Multinational firms are larger on average (higher employment, higher sales, more capital intensive, more productive).

UK based plants (means 96-00)	Foreign Owned	National
Number of observations	3499	161234
Value added/employee	44.6	27.98
Output/employee	151.98	76.52
Mean Employment	485.05	142.09
Capital/employee	98.82	38.23
Intermediate inputs/employee	107.81	50.52

Source: Criscuolo and Martin '09, table 4

Productivity is Important: MNEs Pay Higher Wages

- Almost all studies (out of many!) find that MNEs pay higher wages (see review of Harrison, Rodriguez-Clare 10)
  - Premiums range from 10-20% (US, UK) to 40-50% (Hungary, Brazil).
  - Sorting to industries with high average output (e.g. cars, planes) can partially explain it.

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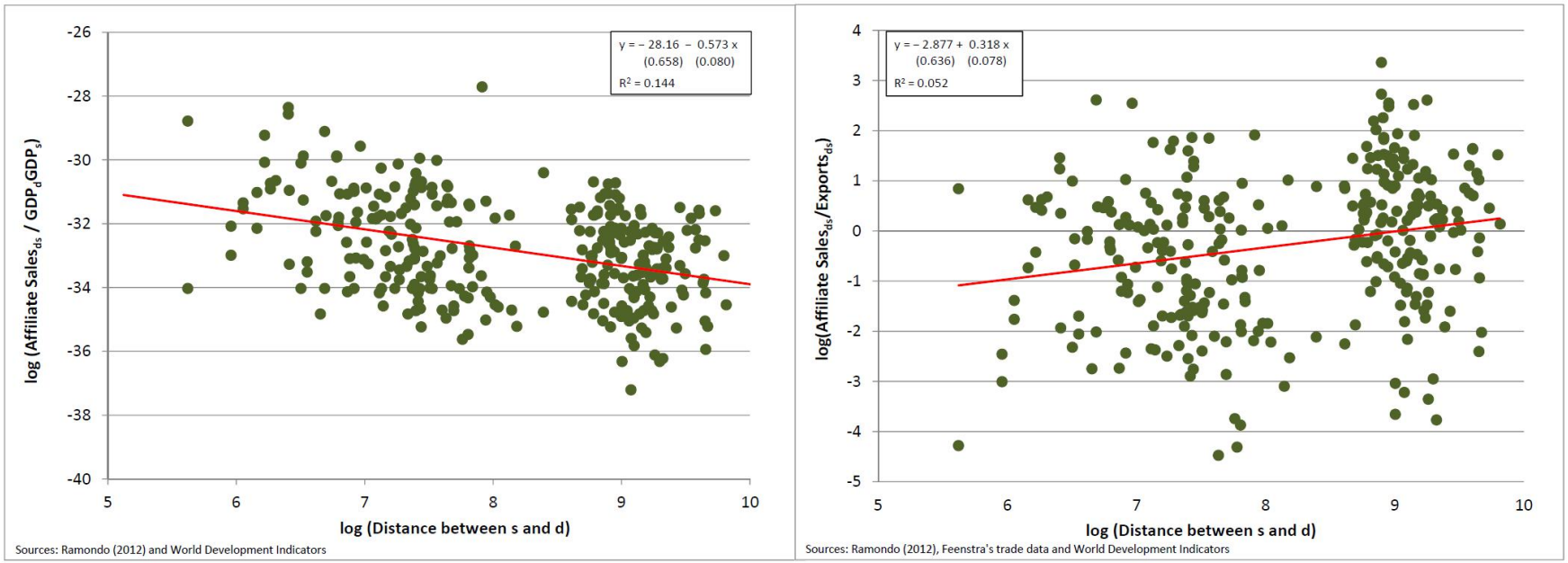
- Conyon et. al. (2002) find (for UK) that:
  - Wages increase by 3.4% after acquisition of domestic companies by foreign investors;
  - Wages decline by 2.1% after acquisition of foreign-owned companies by domestic investors.
  - Some more reliable estimates for the effects of multinationals on wages.

Distance is Critical

- Multinational Activity and Multinational's export both fall with distance but in different rates



Affiliates sales and Exports of US multinationals from the US (s) to various destinations (d).
Source: Antras, Yeaple '14

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A Simple Model to Compute the Gains from Trade

- From A Primer on Gains from Openness (Arkolakis '15)
- Assumptions:
  - 2 countries. Country 1 produces good 1, country 2 produces good 2.
    - Now, country 2 can produce good 2 in either country 1 or in country 2
  - Representative consumer in each country
  - Perfect competition

Firms

- Firms produce the good of their country using labor
- Iceberg trade costs τ_{12} and τ_{21} if the good is exported
- MNE cost γ_{12}, γ_{21} if a good is produced abroad
- Prices for country 1, where we denote a good produced by a MNE with an M:
  - domestically produced goods: $p_{11} = w_1$, $p_{21}^M = \gamma_{21} w_1$
  - foreign produced goods $p_{21} = w_2 \tau_{21}$, $p_{11}^M = \gamma_{12} w_2 \tau_{21}$

Consumers

- Representative consumer with Constant Elasticity of Substitution preferences over the four goods:

$$U_1(c_1, c_2, c_1^M, c_2^M) = \left((c_1)^{\frac{\sigma-1}{\sigma}} + (c_2)^{\frac{\sigma-1}{\sigma}} + \left(c_1^M\right)^{\frac{\sigma-1}{\sigma}} + \left(c_2^M\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

  - c_1: consumption of the home good produced at home by the home consumer
  - c_1^M: consumption of the home good produced abroad by the home consumer
  - c_2: consumption of the foreign good produced abroad by the home consumer
  - c_2^M: consumption of the foreign good produced at home by the home consumer
  - σ: the elasticity of substitution across the two varieties

Gains from MP

- Following the same logic as in the previous model, we have

$$\tilde{\lambda} = \frac{p_{11}^{1-\sigma}}{P_1^{1-\sigma}}, \;\; P_1^{1-\sigma} = (p_1)^{1-\sigma} + (p_1^M)^{1-\sigma} + (p_2)^{1-\sigma} + (p_2^M)^{1-\sigma}$$

  where $\tilde{\lambda}$ here is the consumption share of the home good produced at home

- Note that $\tilde{\lambda}$ is smaller than the share of goods produced at home, λ (since both home and foreign goods can be produced at home)

- Notice that $p_{11} = w_1/A_1$, so that welfare is given by

$$W^M = \frac{w_1}{P_1} = A_1 \tilde{\lambda}^{1/(1-\sigma)}$$

Gains from MP

- Profound significance of this result: gains from openness depend also on multinational activity since if $\sigma > 1$

$$W^M = \tilde{\lambda}^{1/(1-\sigma)} > \lambda^{1/(1-\sigma)}$$

- Notice: from 2nd graph in this lecture, share of foreign firms production in US is in 2005 is up to 20 percentage points ($\tilde{\lambda} \approx 0.78$)
  - Gains $0.78^{-1/5} = 1.05$ almost triple than the gains from only trade!

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Appendix: Constructing λ_{jj}

- Using World Input-Output Data, compute the share of intermediate production done abroad for each country.
- Using World Bank data, scale import share of GDP by the previous to compute value-added abroad
- Subtract previous from 1 to compute λ_{jj}

Import Shares and Export Shares

- Import shares are given by

$$\lambda_{ij} = \frac{X_{ij}}{\sum_k X_{kj}} = \frac{\left(\frac{w_i}{A_i}\tau_{ij}\right)^{1-\sigma}}{\sum_k \left(\frac{w_k}{A_k}\tau_{kj}\right)^{1-\sigma}}$$

- Export shares (i.e. the share of output going to a given country) is

$$y_{ji} = \frac{X_{ji}}{\sum_k X_{jk}} = \frac{\frac{(w_j\tau_{ji}/A_j)^{1-\sigma}}{P_i^{1-\sigma}} L_i w_i}{\sum_k \frac{(w_j\tau_{jk}/A_j)^{1-\sigma}}{P_k^{1-\sigma}} L_k w_k}$$

Direct and Indirect Effects of the Shock

- Using $P_j = \frac{w_j}{W_j}$, $L_j P_j^{\sigma-1} w_j = A_j^{\sigma-1} w_j^{1-\sigma}$, and (1) and denoting $\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1} < 1$:

$$w_j^{\sigma} L_j = \sum_k \left(\frac{\tau_{jk}}{A_j} \right)^{1-\sigma} P_k^{\sigma-1} L_k w_k \iff$$

$$\frac{1}{L_j^{\sigma/(2\sigma-1)}} \left(A_j W_j\right)^{\frac{\sigma-1}{2\sigma-1}\sigma} L_j = \sum_k \left(\frac{\tau_{jk}}{A_j} \right)^{1-\sigma} A_k^{\sigma-1} \left(\frac{1}{L_k^{1/(2\sigma-1)}} \left(A_k W_k\right)^{\frac{\sigma-1}{2\sigma-1}} \right)^{1-\sigma} \iff$$

$$\left(W_j\right)^{\tilde{\sigma}\sigma} L_j^{\tilde{\sigma}} = A_j^{\tilde{\sigma}(\sigma-1)} \sum_k L_k^{\tilde{\sigma}} \left(\tau_{jk}\right)^{1-\sigma} A_k^{(\tilde{\sigma}-1)(1-\sigma)} \left(W_k\right)^{\tilde{\sigma}(1-\sigma)}$$

- Solution for all $j = 1, ..., N$ (recall $\sigma > 1 \iff \tilde{\sigma} < 1$, $\frac{1-\tilde{\sigma}}{\tilde{\sigma}} = \frac{\sigma}{\sigma-1}$)

$$d \log W_j + \frac{\sigma-1}{\sigma} \sum_k y_{jk} d \log W_k = \frac{\sigma-1}{\sigma} \sum_k y_{jk} \left[\underbrace{d \log \frac{A_j}{\tau_{jk}}}_{\text{productivity effect}} + \underbrace{\frac{1-\tilde{\sigma}}{\tilde{\sigma}} d \log A_k}_{\text{terms-of-trade effect}} \right]$$

  - Consider a productivity shock in one country, i.e. $d \log A_k$

Appendix: Analyzing the China Shock

- In the next few slides, we provide further details on the analysis of China's productivity shock (and its effects)

- Start with the equation from the previous slide:

$$d \log W_j + \frac{\sigma - 1}{\sigma} \sum_k y_{jk} d \log W_k = \frac{\sigma - 1}{\sigma} y_{jC} \left[d \log A_C \times \mathbb{I}_{j=C} + \frac{1 - \tilde{\sigma}}{\tilde{\sigma}} d \log A_C \right]$$

  - where $\mathbb{I}_{j=C}$ is and indicator function that turns ''on'' whenever $j = C$

- **(1) Productivity effect**: $d \log A_C \times \mathbb{I}_{j=C}$ only ''lights up'' when equation is evaluated at $j = C$
  - Direct productivity effect only on Chinese welfare ($d \log W_C$)
  - But indirect productivity effect on other countries' welfares too ($\lambda_{jk} d \log W_k$)

- **(2) Terms-of-trade effect**:
  - Direct terms-of-trade effect on all countries' welfares ($\lambda_{jC} \frac{1-\tilde{\sigma}}{\tilde{\sigma}} d \log A_C$)
  - Indirect terms-of-trade effect on all countries' welfares ($\lambda_{kj} d \log W_k$)

Multiple Countries Multiple Shocks

- We have

$$(W_j)^{\tilde{\sigma}\sigma} L_j^{\tilde{\sigma}} = A_j^{\tilde{\sigma}(\sigma-1)} \sum_k L_k^{\tilde{\sigma}} (\tau_{jk})^{1-\sigma} A_k^{(\tilde{\sigma}-1)(1-\sigma)} (W_k)^{\tilde{\sigma}(1-\sigma)}$$

- Totally differentiate this equation. Also note: $d \ln x = \frac{dx}{x}$, $d \sum_i x_i = \sum_i dx_i$,

$$\frac{L_k^{\tilde{\sigma}} \left(\frac{A_k^{\tilde{\sigma}-1}}{A_j^{\tilde{\sigma}}} \tau_{jk} \right)^{1-\sigma} (W_k)^{\tilde{\sigma}(1-\sigma)}}{L_j^{\tilde{\sigma}} W_j^{\tilde{\sigma}\sigma}} = \frac{L_k^{\tilde{\sigma}} \left(\frac{A_k^{\tilde{\sigma}-1}}{A_j^{\tilde{\sigma}}} \tau_{jk} \right)^{1-\sigma} (W_k)^{\tilde{\sigma}(1-\sigma)}}{\sum_m L_m^{\tilde{\sigma}} \left(\frac{A_m^{\tilde{\sigma}-1}}{A_j^{\tilde{\sigma}}} \tau_{jm} \right)^{1-\sigma} (W_m)^{\tilde{\sigma}(1-\sigma)}} = y_{jk}$$

- We obtain:

$$\tilde{\sigma}\sigma d \log W_j = \sum_k y_{jk} \left[\tilde{\sigma}(1-\sigma) d \log W_k + (1-\sigma) d \log \tau_{jk} + (\sigma-1) \left(\tilde{\sigma} d \log A_j + (1-\tilde{\sigma}) d \log A_k \right) \right] \Longleftrightarrow$$

$$d \log W_j + \frac{\sigma-1}{\sigma} \sum_k y_{jk} d \log W_k = \frac{\sigma-1}{\sigma} \sum_k y_{jk} \left[\underbrace{d \log \frac{A_j}{\tau_{jk}}}_{\text{productivity effect}} + \underbrace{\frac{1-\tilde{\sigma}}{\tilde{\sigma}} d \log A_k}_{\text{terms-of-trade effect}} \right]$$