The Economics of Space 433: Lectures 3 and 4

A Brief History of Space in Economics

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In Economics, Space has History

- Spatial economics is one of the oldest and most celebrated fields in economics
  - Models as old as the work of Ricardo 1817, Von Thunen 1826, Hotelling 1929 have fascinated scholars of economics since the 19th century
  - Ideas of spatial agglomeration through knowledge spillovers were introduced more than 100 years ago (Marshall 1920)

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- We will illustrate some of the key ideas in these theories to understand the role of space in traditional models
  - The Von Thunen model of the Central Business District
  - The Hotelling model of location choice.
  - The workhorse spatial setup of Rosen Roback '82
  - Instead of the traditional Ricardian theory of comparative advantage we will go over the Armington theory of product differentiation (Anderson '79)

- In lectures 5-6, we delve into the concepts of agglomeration economies and amenity spillovers

Main Elements of a Spatial Model

- We discussed the main elements of a spatial model
  - Topography: productivities (A_i), amenities (u_i), and spatial links (τ_{ij})
  - Geography: economic fundamentals $\left(\bar{A}_i, \bar{u}_i\right)$ that determine these overall productivities/amenities, and transportation network $\bar{t}_{ij}$
  - We make use of these concepts throughout the course

- We now present a brief history of the key ideas based on which spatial economics were formed

Roadmap

- **The Monocentric City Model**
- The Rosen-Roback Spatial Framework
- Hotelling Location Model
- The Armington Model

The Monocentric City Model

- A remarkable first approach to spatial analysis pioneered by Von Thunen 1826
  - The model was designed to capture the specialization of production in cities
  - A farming sector that locates in the suburbs produces to sell to the downtown
  - The downtown produces manufacturing that is shipped to the suburbs

- How further out does the city extend?
  - If transportation is costly, with a homogeneous product, there is a finite threshold that defines the border of the city
  - We will set this up on the line as in the original model

Setup

- Sectors: manufacturing M and agriculture A each producing a single good
- Space:
  - c is the city center. f is the agricultural frontier location, $c, f \in S$
  - $\tau_{j,s}$ is the iceberg trade cost from city center c to location j and from j to c of good $s = M, A$
  - We assume that $\tau_{j,s}$ is continuous and monotonically increasing in the distance from c
- Workers: Freely mobile in space (across $j \in S$) and across sectors ($s = M, A$)

Production (Manufacturing)

- M producer:
  - Only locates in city center c and produces a homogeneous manufacturing output
  - Inputs one unit of labor and outputs one unit. Thus $w_{c,M} = p_{c,M}$.

- Due to the trade costs, the price of manufacturing good in location j is

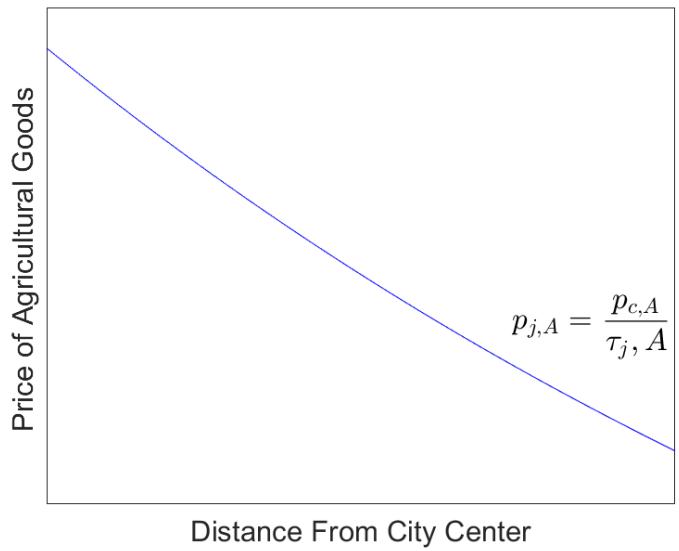
$$p_{j,M} = w_{c,M}\tau_{j,M}$$

Production in Agriculture

- A producer (landlord):
  - Produces a homogeneous agricultural output
  - Inputs $1/\overline{A}_{i,A}$ units of labor and one unit of land and outputs one unit
  - There is only trade between c and each agriculture location. No trade across agriculture locations
- The resulting price is

$$p_{c,A} = p_{j,A}\tau_{j,A} \implies p_{j,A} = p_{c,A}/\tau_{j,A}$$

  - The agricultural price varies across space



Production in Agriculture

- Landlords cannot move
- Using the previous equation the Landlord's problem to maximize rents at j (see TF session for details on profit maximization) is written

$$r_{j,A} = \max\left\{0, p_{j,A} - \frac{w_{j,A}}{\overline{A}_{j,A}}\right\} = \max\left\{0, \frac{p_{c,A}}{\tau_{j,A}} - \frac{w_{j,A}}{\overline{A}_{j,A}}\right\}.$$

  - Agriculture takes place at $j \in S$ where $S \equiv \{j | r_{j,A} \geq 0\}$.
  - At the endogenous agricultural frontier f, it must be that zero rents are made, $r_{f,A} = 0 \implies$

$$\frac{p_{c,A}}{\tau_{f,A}} = \frac{w_{f,A}}{\overline{A}_{f,A}}. \tag{1}$$

Consumer Demand

- Consumers are homogeneous workers
  - Free mobility across sectors, M, A, and locations,j
- Consume M and A to maximize a Cobb-Douglas aggregate of the two
  - $\mu \in (0, 1)$ is a share of manufacturing in consumption
  - Price of the bundle for the consumer, $(p_{j,M})^{\mu} (p_{j,A})^{1-\mu}$
  - (See TF session for details on consumer optimization)
- Workers choose the location j and sector s
  - There are no amenity differences, so $\overline{u}_i = 1$
  - Welfare of the worker at location j and sector s is $W_{j,s} = \frac{w_{j,s}}{\left(p_{j,M}\right)^{\mu}\left(p_{j,A}\right)^{1-\mu}}$
  - In equilibrium (due to mobility), indifferent across sectors and locations i.e. $W_{j,s} = \bar{W}$

Agricultural Wage in Space

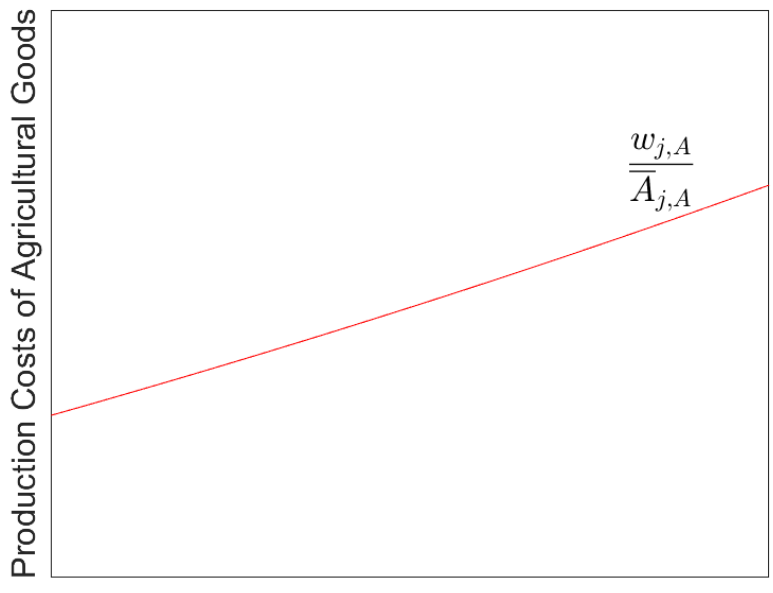
- We have

$$\bar{W} = \frac{w_{j,A}}{(p_{j,M})^{\mu}(p_{j,A})^{1-\mu}} = \frac{w_{j,A}}{(p_{c,M}\tau_{j,M})^{\mu}\left(\frac{p_{c,A}}{\tau_{j,A}}\right)^{1-\mu}}$$

  so that

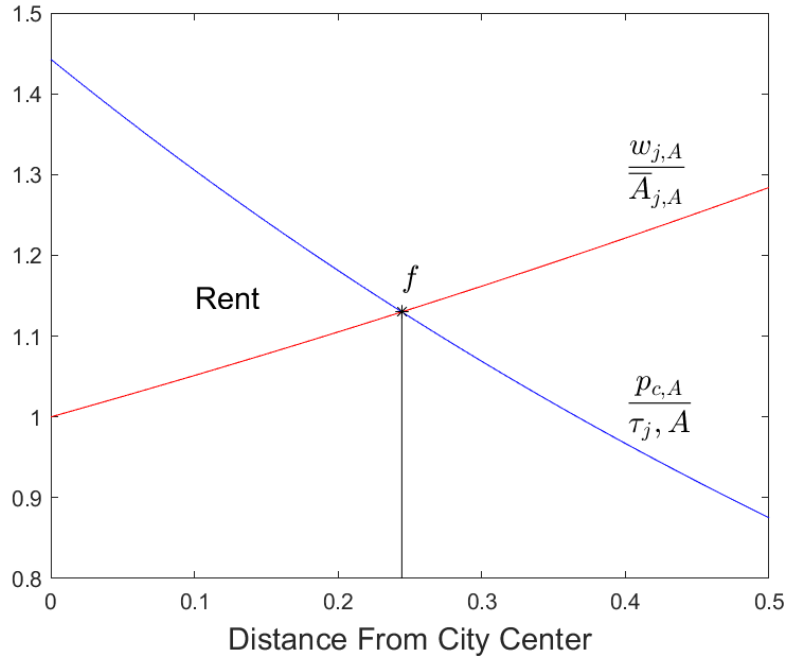
$$w_{j,A} = (p_{c,M}\tau_{j,M})^{\mu}\left(\frac{p_{c,A}}{\tau_{j,A}}\right)^{1-\mu}\bar{W}$$

- If $\frac{\tau_{j,M}^{\mu}\tau_{j,A}^{\mu-1}}{\bar{A}_{j,A}}$ is increasing in the distance from the center, $\frac{w_{j,A}}{\bar{A}_{j,A}}$ is increasing



Rents in Space and City Frontier

We now use (1) to determine the frontier of the city



- Agricultural goods price $p_{j,A}$ and production cost $\frac{w_{j,A}}{\overline{A}_{j,A}}$ at each location as a function of the distance from the center, d_j
- If $p_{j,A}$ decreases and $\frac{w_{j,A}}{\overline{A}_{j,A}}$ increases in the distance, we have a linear city $[0, f]$
- Assume $\overline{A}_{j,A} = 1$, $\tau_{j,s} = e^{\tau_s d_j}$, $\tau_M = 2, \tau_A = 1, \mu = 0.5$, total population $N = 1$
- Note: $p_{c,A}$ is an equilibrium object to be solved later

Equilibrium

- Set the numeraire $p_{c,M} = 1 \implies w_{c,M} = p_{c,M} = 1$
  - We want to solve for the price $p_{c,A}$, the (endogenous) location of the frontier f, and the level of welfare $\bar{W}$

- The equilibrium conditions boil down to equation (1), free mobility, and agriculture market clearing:

$$W_{c,M} = W_{j,A} = \bar{W}, \tag{2}$$

$$\underbrace{(1-\mu)\, Y_c}_{A \text{ spending in } c} = \sum_{i \in S} X_{ic,A}. \tag{3}$$

  where $X_{ic,A}$ is the imports of agriculture goods at the city center from region i, and Y_c is total income in the city center (from manufacturing sales)

- We need to use (2) and (3) along with (1) to solve for the full equilibrium
  - (More details in the TA session)

Equilibrium Characterization: Welfare Equalization

- We can use equation (2) and (1) to solve for $p_{c,A}$ as a function of the location of the agriculture frontier, f
- Equation (2), free mobility between manufacturing workers in the city center and agriculture workers in the frontier, dictates

$$W_{c,M} = \frac{w_{c,M}}{(p_{c,M})^{\mu}(p_{c,A})^{1-\mu}} = \frac{w_{f,A}}{(p_{c,M}\tau_{f,M})^{\mu}(p_{c,A}/\tau_{f,A})^{1-\mu}} = W_{f,A}$$

$$\overset{w_{c,M}=p_{c,M}=1}{\Longrightarrow} 1 = \tau_{f,A}(\tau_{f,A}\tau_{f,M})^{-\mu} w_{f,A}$$

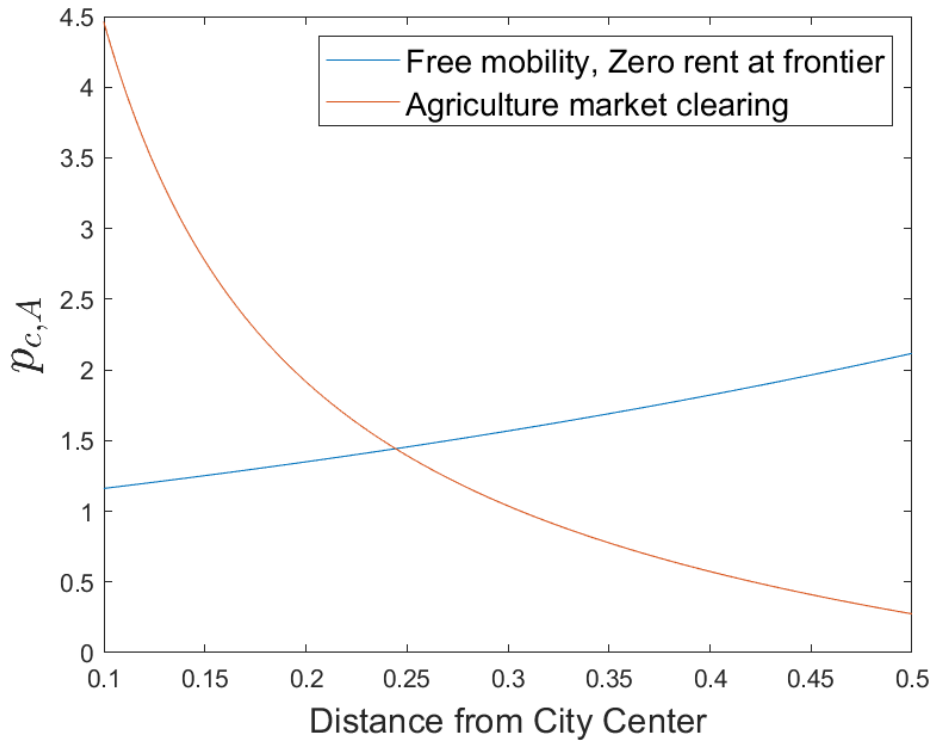
- Combining this equation with (1), $\frac{p_{c,A}}{\tau_{f,A}} = \frac{w_{f,A}}{\overline{A}_{f,A}}$, we obtain

$$p_{c,A} = \frac{(\tau_{f,M})^{\mu}(\tau_{f,A})^{\mu}}{\overline{A}_{f,A}}. \tag{4}$$

  - Which also gives us all the other prices since $p_{i,A} = \frac{p_{c,A}}{\tau_{i,A}}$

Linear City

- Using this last result, we can finally solve for f
  - Plot equation (4) (free mobility combined with zero rent)
  - Agriculture market clearing (3) (again, see details in TF session)
- Their intersection determines the location of frontier f and agriculture price at the center $p_{c,A}$



Modern Reformulations with Commuting

- The model has been recently reformulated by Alonso '64 and Muth '69 as a commuting model
  - Cost of commuting to travel downtown in terms of a time endowment
  - Analysis of the modern city

- A recent quantitative application of this reformulation was developed by Ahlfeldt, Redding, Sturm, Wolf '15
  - To analyze the actual commuting patterns and the city economic activity with a wealth of data from Berlin

Roadmap

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Baseline Urban Model

- We now study the Rosen-Roback '82 model
  - In this model, a single homogenous agricultural good is freely traded
  - This means that prices are the same everywhere
- A mere trick to price labor: there is no trade in equilibrium
  - We will relax this assumption down the road
- Denote by j or i the location of the worker and the firm

Workers

- Worker utility

$$W_j = C_j u_j$$

where C_j is the consumption in location j and u_j is the amenity

- Agent has a budget constraint

$$p_j C_j = w_j$$

where p_j is price of agricultural good, C_j is consumption, and w_j is wage in j
  - Good is freely traded so that $p_j = p$. We can normalize one price, $p_j = p = 1$
  - Thus, $C_j = w_j$

Producers

- Perfectly competitive producers with labor productivity A_i
- Given no trade costs, we have price equalization

$$p = w_i/A_i \implies w_i = A_i$$

Productivities and Amenities

- Let us now consider the case that

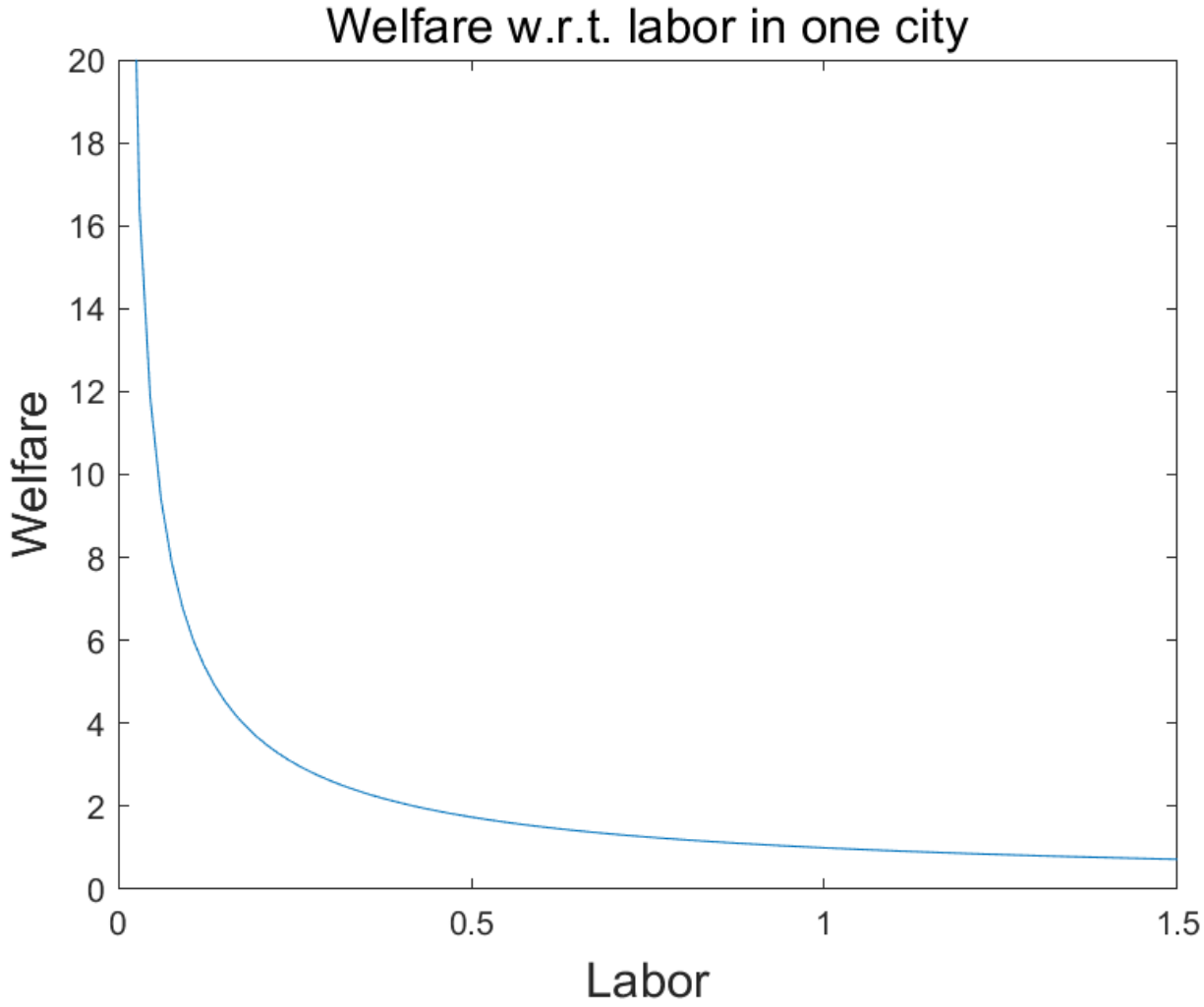
$$A_i = \bar{A}_i L_i^{\alpha}$$
$$u_i = \bar{u}_i L_i^{-\beta}$$

  where α controls strength of the agglomeration forces and β strength of dispersion forces
  - i.e. how much A_i and u_i change when population doubles
- Thus, taking all into account

$$W_j = w_j u_j = A_j u_j = \bar{A}_j \bar{u}_j L_j^{\alpha-\beta}$$

- Assume $\beta > \alpha$ so that model is 'well-behaved': welfare decreases with population
  - If $\alpha > \beta$ possibility of 'black hole'! All population may end up in one location
  - Thus, $\beta > \alpha$ is also sometimes referred to as a no-'black hole' condition

Welfare and Population in Our Baseline Model



Labor Supply and Welfare

- We assume that workers are freely mobile and thus in equilibrium $W_j = W$
- Now notice that we have

$$W_j = A_j u_j = \bar{A}_j \bar{u}_j L_j^{\alpha-\beta} \implies L_j = \left(\frac{W_j}{\bar{A}_j \bar{u}_j}\right)^{\frac{1}{\alpha-\beta}} \tag{5}$$

  - Since $W_j = W \; \forall \; j$, we also obtain

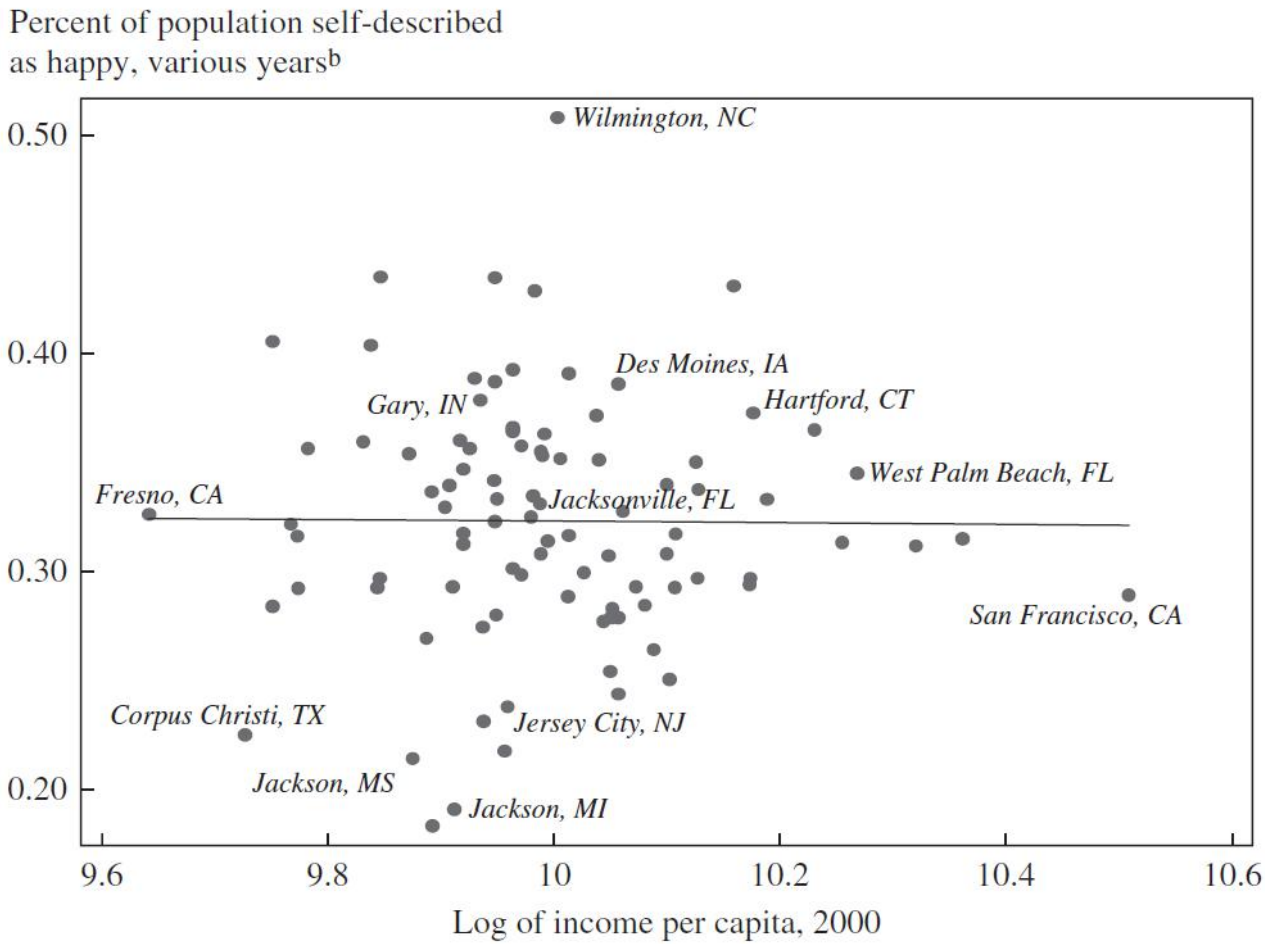
$$L_j = (\bar{A}_j \bar{u}_j)^{\frac{1}{\beta-\alpha}} W^{\frac{1}{\alpha-\beta}} \tag{6}$$

- Summing across locations,

$$\bar{L} = \sum_j L_j = W^{1/(\alpha-\beta)} \sum_j \left(\frac{1}{\bar{A}_j \bar{u}_j}\right)^{\frac{1}{\alpha-\beta}} \implies$$

$$W = \frac{\left[\sum_j (\bar{A}_j \bar{u}_j)^{\frac{1}{\beta-\alpha}}\right]^{\beta-\alpha}}{\bar{L}^{\beta-a}} \tag{7}$$

Evidence from Glaeser Gottlieb '08: Welfare Equalization



Sources: U.S. Census Bureau, General Social Survey 1972–2006. See appendix B for details.
a. Each observation represents a single metropolitan statistical area under the 2006 definitions.
b. Respondents are considered happy if they chose the happiest of three possible responses.

Labor Allocations

- Combining 6 and 7 we obtain the population in terms of fundamentals

$$L_j = \frac{(\bar{A}_j \bar{u}_j)^{\frac{1}{\beta-\alpha}}}{\sum_{j'} (\bar{A}_{j'} \bar{u}_{j'})^{\frac{1}{\beta-\alpha}}} \bar{L}$$

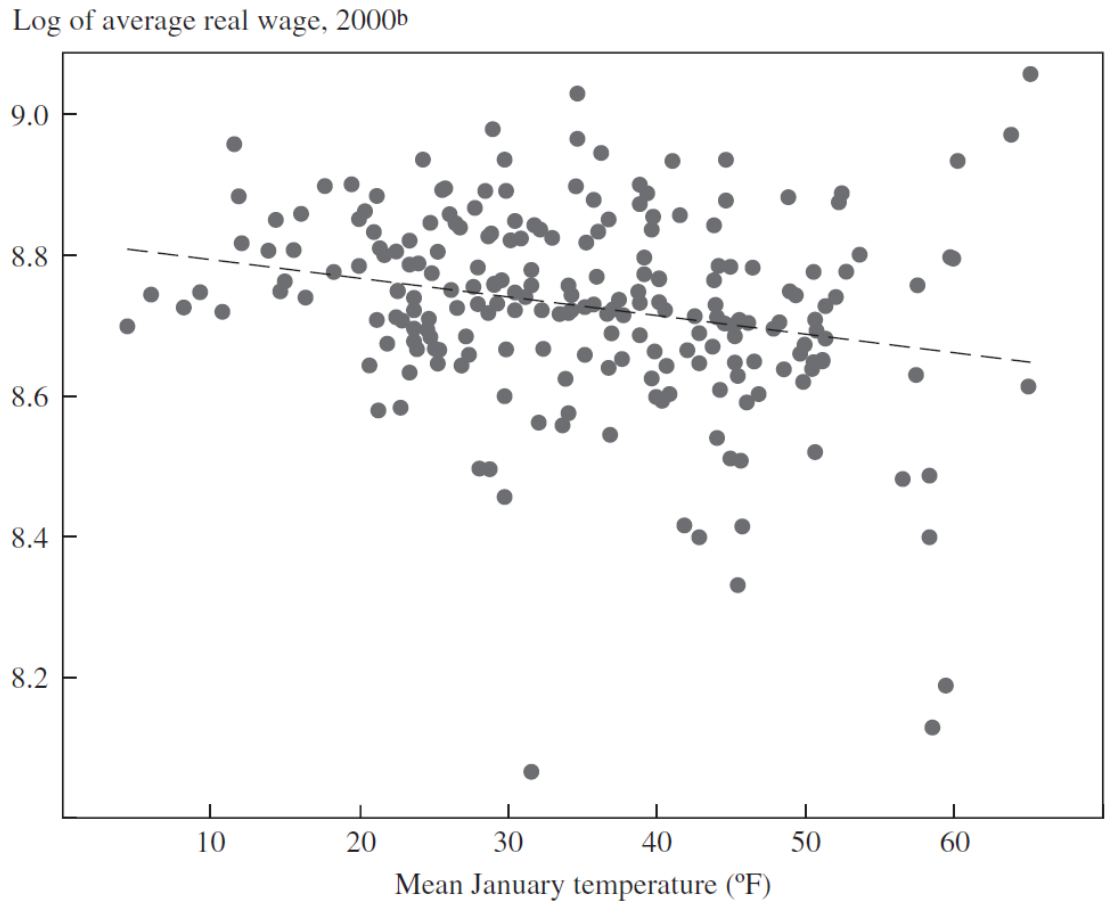
- Taking logs of equation (6) gives,

$$\log L_j = \frac{1}{\beta-\alpha} \log \bar{A}_j + \frac{1}{\beta-\alpha} \log \bar{u}_j + \frac{1}{\alpha-\beta} \log W$$

The Rosen-Roback '82 model: Predictions

- Let us assume $\beta > \alpha$,and keep W fixed
- More workers in j if (ceteris paribus):
  - More productive $\iff$ $\bar{A}_j$ higher
  - Better amenities $\iff$ $\bar{u}_j$ higher
- What about w_j? Recall $W = w_j u_j \iff w_j = W \bar{u}_j^{-1} L_j^{\beta}$
  - To a first order it should be decreasing in amenities
  - But it also depends on the equilibrium allocation of L_j ...
  - ...and thus the correlation of $\bar{A}_j$ and $\bar{u}_j$ in space

Evidence from Glaeser Gottlieb '08: Wages and Amenities



Sources: U.S. Census Bureau; American Chamber of Commerce Research Association; City and County Data Book, 1994. See appendix B for details.

a. Each observation represents a single metropolitan statistical area under the 1999 definitions, using primary rather than consolidated MSAs where applicable and New England county metropolitan areas where applicable.

b. Real wages are defined as the ratio of income per capita to the ACCRA cost of living index.

Main Insights and Critiques

- The main insight of the Rosen-Roback framework is to put the agglomeration and dispersion forces in the forefront of the analysis
  - Due to supportive evidence of population sorting on productivities and amenities, it has been an extremely practical setup

- The main limitation is that space, at the end of the day, is modeled in a very limited way
  - No cost of moving goods, no frictions of moving people or firms
  - The next two models, developed independently, discuss the location of firms and the costs of moving goods
  - We will use insights from both setups when we develop the modern spatial approach in future lectures

Roadmap

- The Monocentric City Model
- The Rosen-Roback Spatial Framework
- **Hotelling Location Model**
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Hotelling Model: Linear City

- The Hotelling model studies the endogenous firm location
  - Firms locate taking into account transportation cost to consumers
- Consumers
  - Distributed uniformly on a line $[0, 1]$ (the linear city)
  - Assume that consumers have a unitary demand and a linear transportation cost, t
- Firms
  - Firms sell a homogeneous good for free
  - So, consumers only pay the the transportation cost of getting to a firm

Hotelling Model: Planner Solution

- One way to solve the model is to minimize the total transportation cost of the system with respect to the location of the firms on the line
  - This is the ''social planners'' solution
  - We will study the insights arising from this solution using a 2-firm example

- Intuition: Why shouldn't we locate firms in the middle or at the edge of the line?
  - Locating in the middle gives a firm access to the most consumers
  - But this isn't optimal because we can lower the cost for consumers at the extremes by spreading out firm locations
  - That is, it is possible to choose locations of the firm in a way that makes everyone better or at least as good as they were before

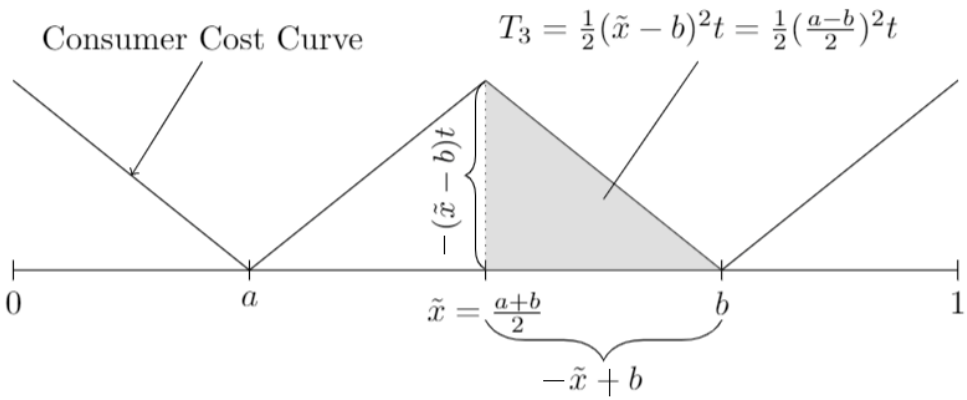
Hotelling Model: Linear City Solution

- The cost curve in the diagram gives the cost a consumer at $x \in [0,1]$ pays
  - Summing over the costs yields the total transportation cost
    - E.g. total cost for consumers between a and $\frac{a+b}{2}$ is volume of triangle

$$T = \frac{1}{2} \underbrace{\left(\frac{a+b}{2} - a\right)}_{\text{base (hor. distance)}} \times \underbrace{t\left(\frac{a+b}{2} - a\right)}_{\text{height (slope x hor. distance)}}$$

  - Planner minimizes transportation cost. Chooses firm locations to minimize volume

- Example with 2 firms located at points a, b (see figure below)
  - $\tilde{x}$ denotes the indifferent consumer.
    - Those to the left will buy from a, while those to the right from b
  - For example, T_3, is total cost for consumers in $(\tilde{x}, b)$
  - Solution is $a = \frac{1}{4}, b = \frac{3}{4}$ with total cost, $\frac{t}{8}$ (see HW)



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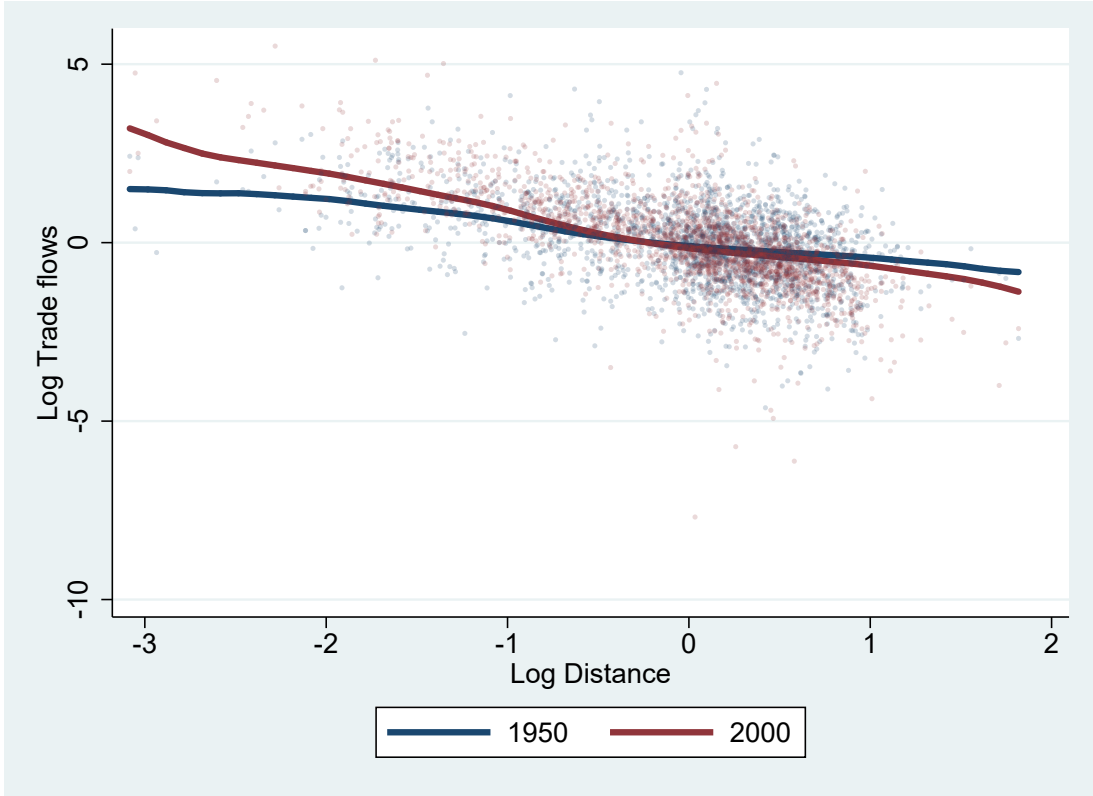
Setup

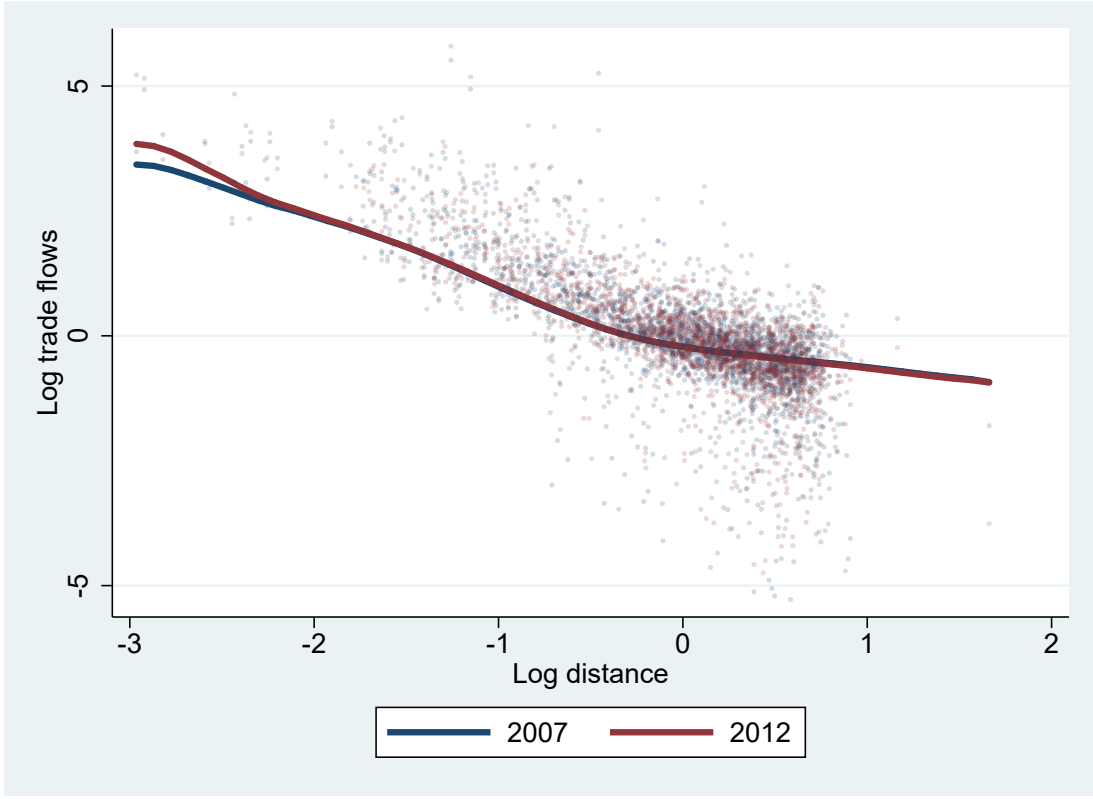
- So far we have assumed that products are homogeneous
- This assumption typically implies absence of two-way trade or the absence of trade altogether
  - There are two striking facts in the data
  - i) we observe a lot of two way trade ii) most of it between nearby regions
- The Armington model answers the question why people across different locations trade

Patterns of Trade Flows and Distance (World)



Source: Allen Arkolakis '18. This figure plots the estimated coefficient of distance in a trade gravity regression with origin-year and destination-year fixed effects over time using trade flows between locations.

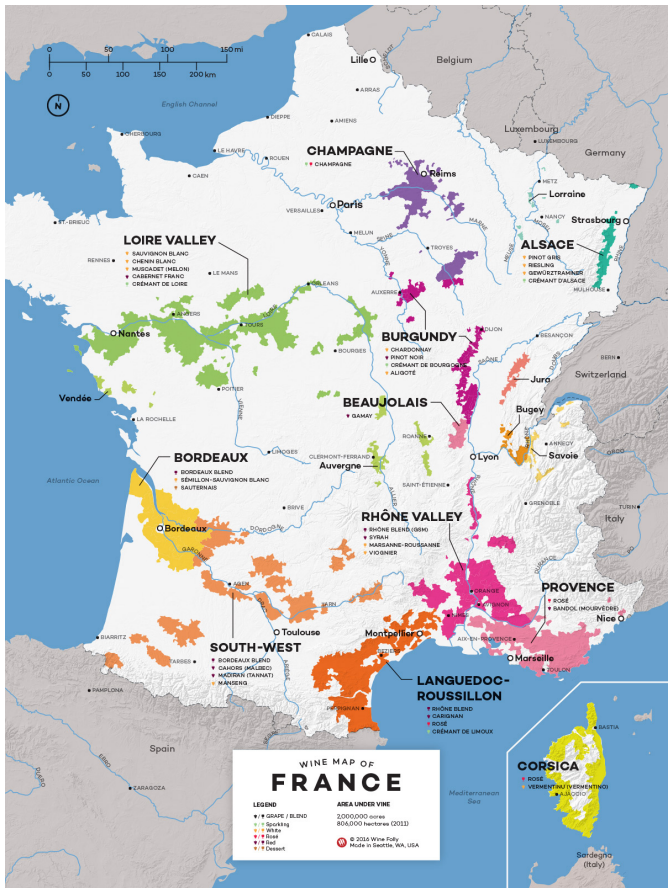
Patterns of Trade Flows and Distance (Within USA)



Source: Allen Arkolakis '18. The figure excludes trade flows within each state. The thick lines are from a nonparametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out the origin-year and destination-year fixed effects.

Setup

- 2 locations denoted by i. Set of locations $S = \{1, 2\}$
  - Each location produces a differentiated commodity
  - E.g. Region-specific varieties of France (see http://winefolly.com/review/french-wine-exploration-map/)



Representative Consumer

- Consumer chooses location j and makes consumption choices

Representative Consumer

- Consumer chooses location j and makes consumption choices
- We assume that the welfare of consumer in location j equals consumption times amenity

$$W_j = C_j \times u_j$$

where C_j is per-capita real consumption in location j and u_j is overall amenity

Representative Consumer: Consumption

- Next, we characterize the representative consumers' consumption choices
- Preferences: Constant Elasticity of Substitution (CES) utility function over two goods, home and foreign

$$C_j(c_{1j}, c_{2j}) = ((c_{1j})^{(\sigma-1)/\sigma} + (c_{2j})^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

  - c_j: consumption of location $j = 1, 2$
  - $\sigma > 0$: elasticity of substitution. For practical purposes, we will henceforth assume that $\sigma > 1$

Representative Consumer: Consumption

- Preferences: Constant Elasticity of Substitution (CES) utility function over two goods, home and foreign

$$C_j(c_{1j}, c_{2j}) = ((c_{1j})^{(\sigma-1)/\sigma} + (c_{2j})^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

- Budget constraint of consumer: $p_{1j}c_{1j} + p_{2j}c_{2j} = w_j$
  - p_{ij}: price of good from location i shipped to j
  - w_j : wages in j

Consumer Optimization: Consumption

- For each location j, consumer maximizes

$$\max_{c_{1i},c_{2i}}((c_{1j})^{(\sigma-1)/\sigma}+(c_{2j})^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

$$\text{subject to } p_{1j}c_{1j}+p_{2j}c_{2j}=w_j$$

- It can be written as the Lagrangian optimization

$$\mathcal{L}=\max_{c_{1i},c_{2i}}((c_{1j})^{(\sigma-1)/\sigma}+(c_{2j})^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

$$+\mu\left(w_j-p_{1j}c_{1j}-p_{2j}c_{2j}\right)$$

Optimal Consumption

- Consumer optimization implies

$$\frac{(c_{1j})^{\frac{\sigma-1}{\sigma}-1}}{(c_{2j})^{\frac{\sigma-1}{\sigma}-1}} = \frac{p_{1j}}{p_{2j}} \implies \frac{(c_{1j})^{-\frac{1}{\sigma}}}{(c_{2j})^{-\frac{1}{\sigma}}} = \frac{p_{1j}}{p_{2j}} \implies$$

$$\frac{c_{1j}}{c_{2j}} = \left(\frac{p_{1j}}{p_{2j}}\right)^{-\sigma} \tag{8}$$

- Using these derivations, we can now define the elasticity of substitution

$$\frac{\partial \ln\left(c_{1j}/c_{2j}\right)}{\partial \ln\left(p_1/p_2\right)} = -\sigma$$

Aggregation: Trade Shares

- An essential derivation is the share of spending in each location's good
- We define the share of location j on goods $i = 1$,

$$\lambda_{1j} = \frac{p_{1j} c_{1j}}{p_{1j} c_{1j} + p_{2j} c_{2j}}$$

  and similar for λ_{2j}

- Recall that from equation (8), we have, $c_{1j}/c_{2j} = (p_{1j}/p_{2j})^{-\sigma}$, so that

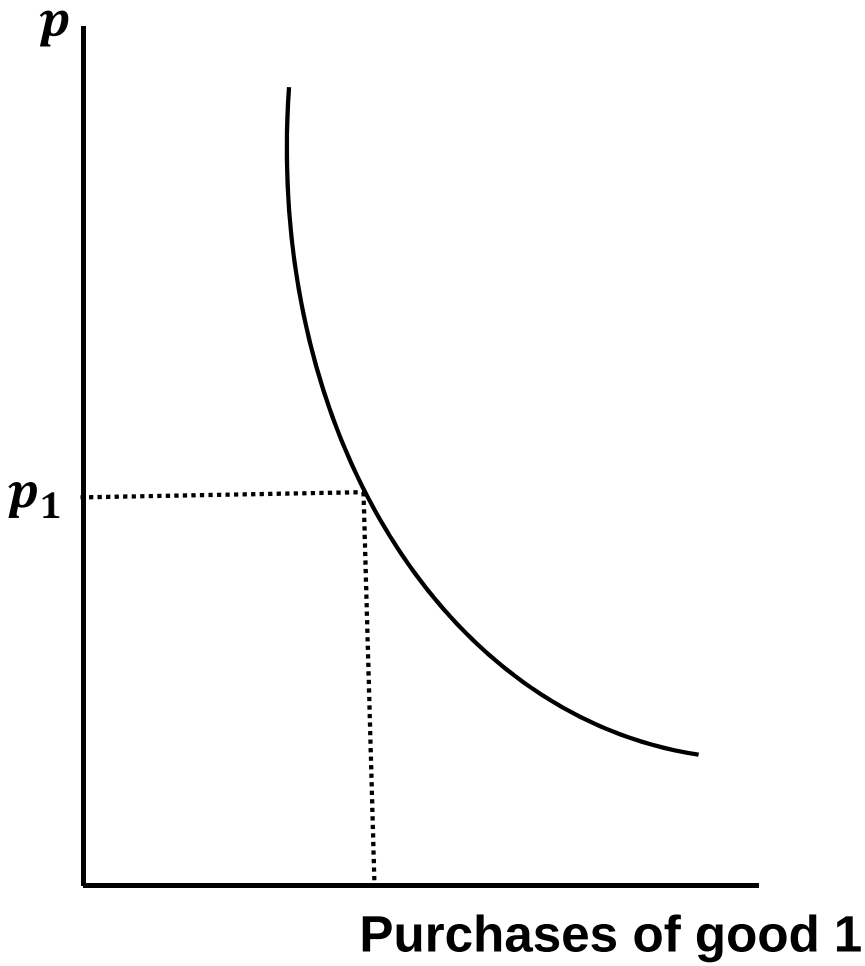
$$\lambda_{1j} = \frac{p_{1j} (p_{1j}/p_{2j})^{-\sigma} c_{2j}}{p_{1j} (p_{1j}/p_{2j})^{-\sigma} c_{2j} + p_{2j} c_{2j}} = \frac{p_{1j} (p_{1j}/p_{2j})^{-\sigma}}{p_{1j} (p_{1j}/p_{2j})^{-\sigma} + p_{2j}} = \frac{p_{1j}^{1-\sigma}}{p_{1j}^{1-\sigma} + p_{2j}^{1-\sigma}} \quad (9)$$

  - Define the Constant Elasticity Price (CES) index P_j

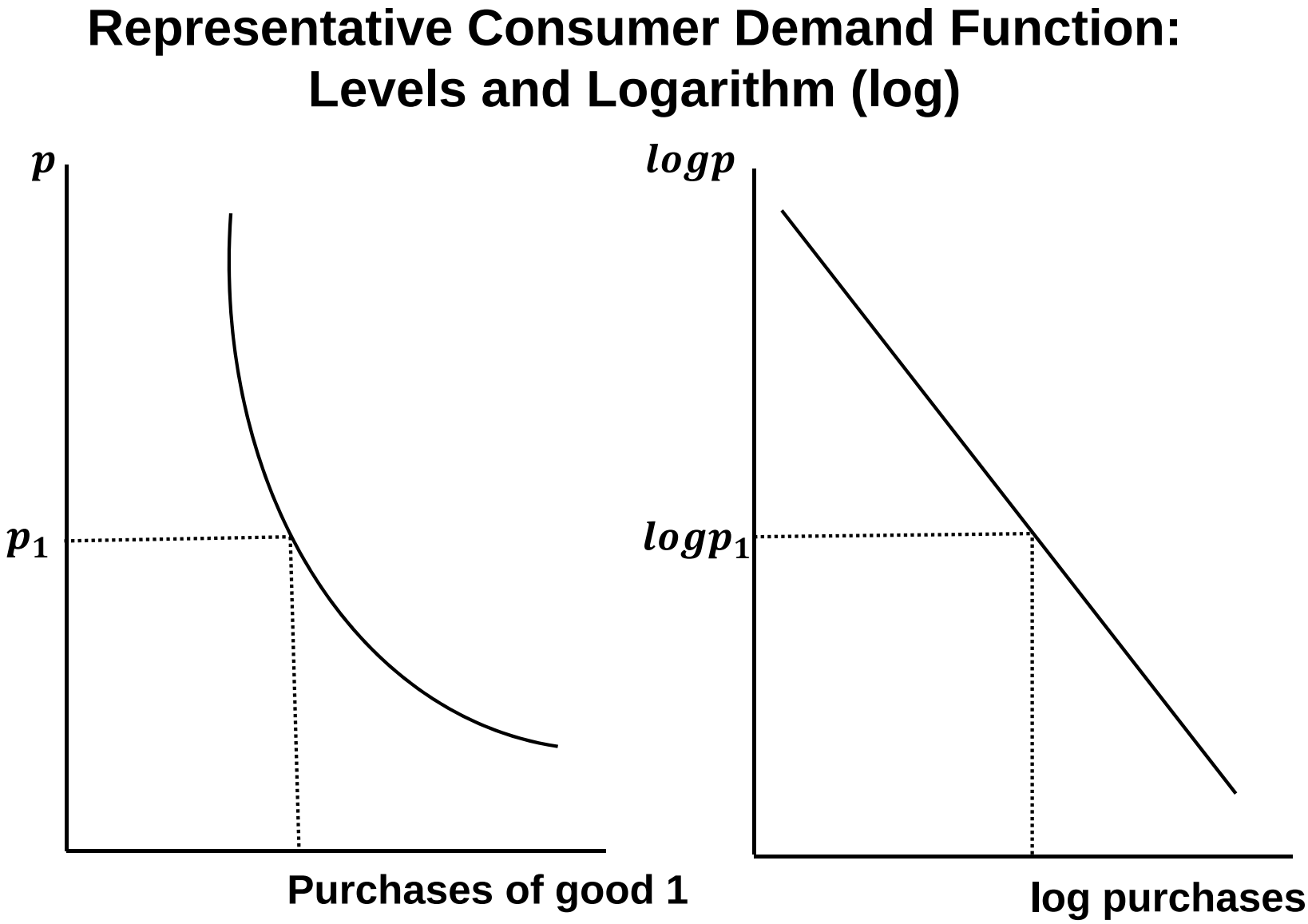
$$P_j^{1-\sigma} \equiv p_{1j}^{1-\sigma} + p_{2j}^{1-\sigma} \quad (10)$$

Demand Function

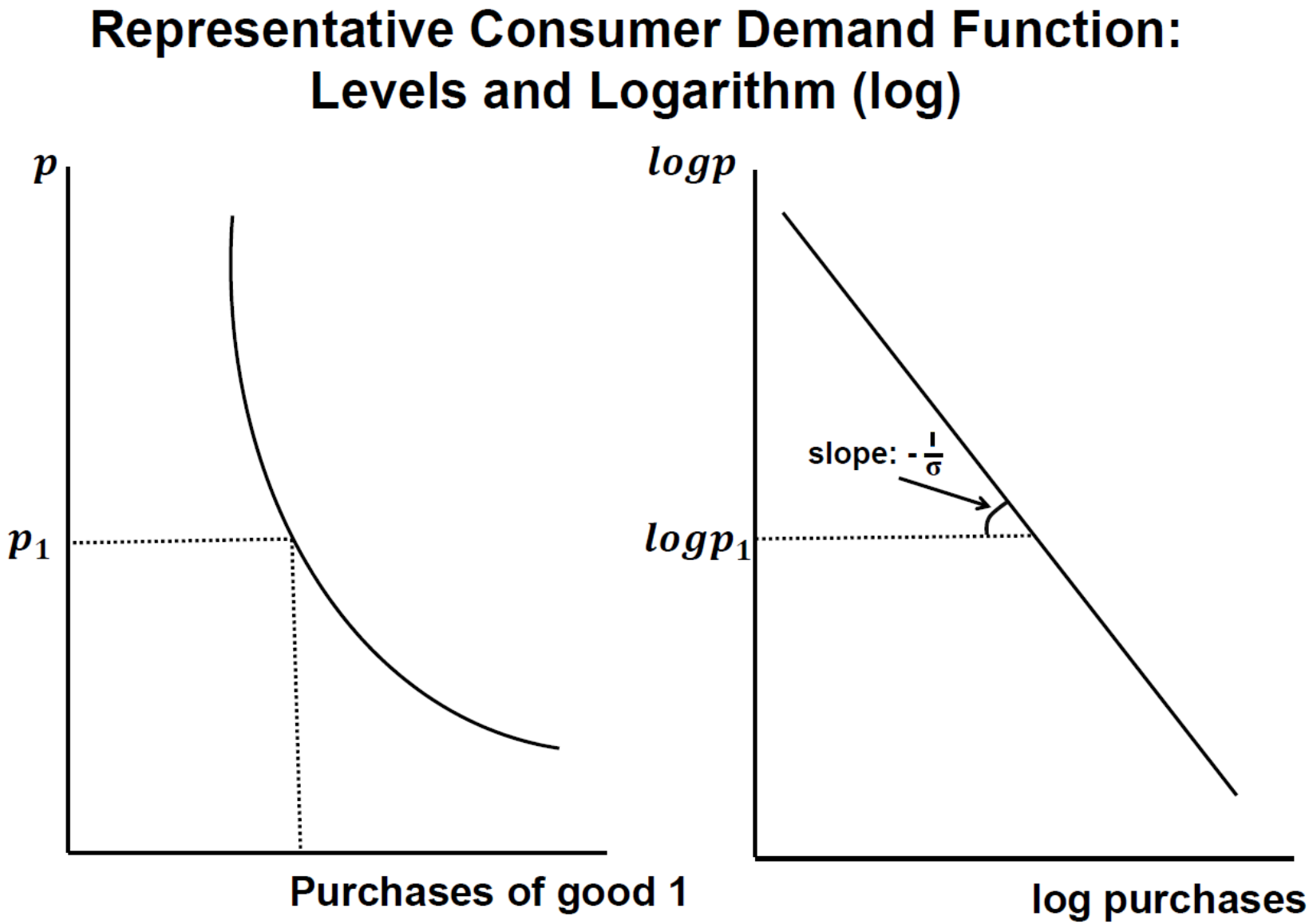
Representative Consumer Demand Function: Levels and Logarithm (log)



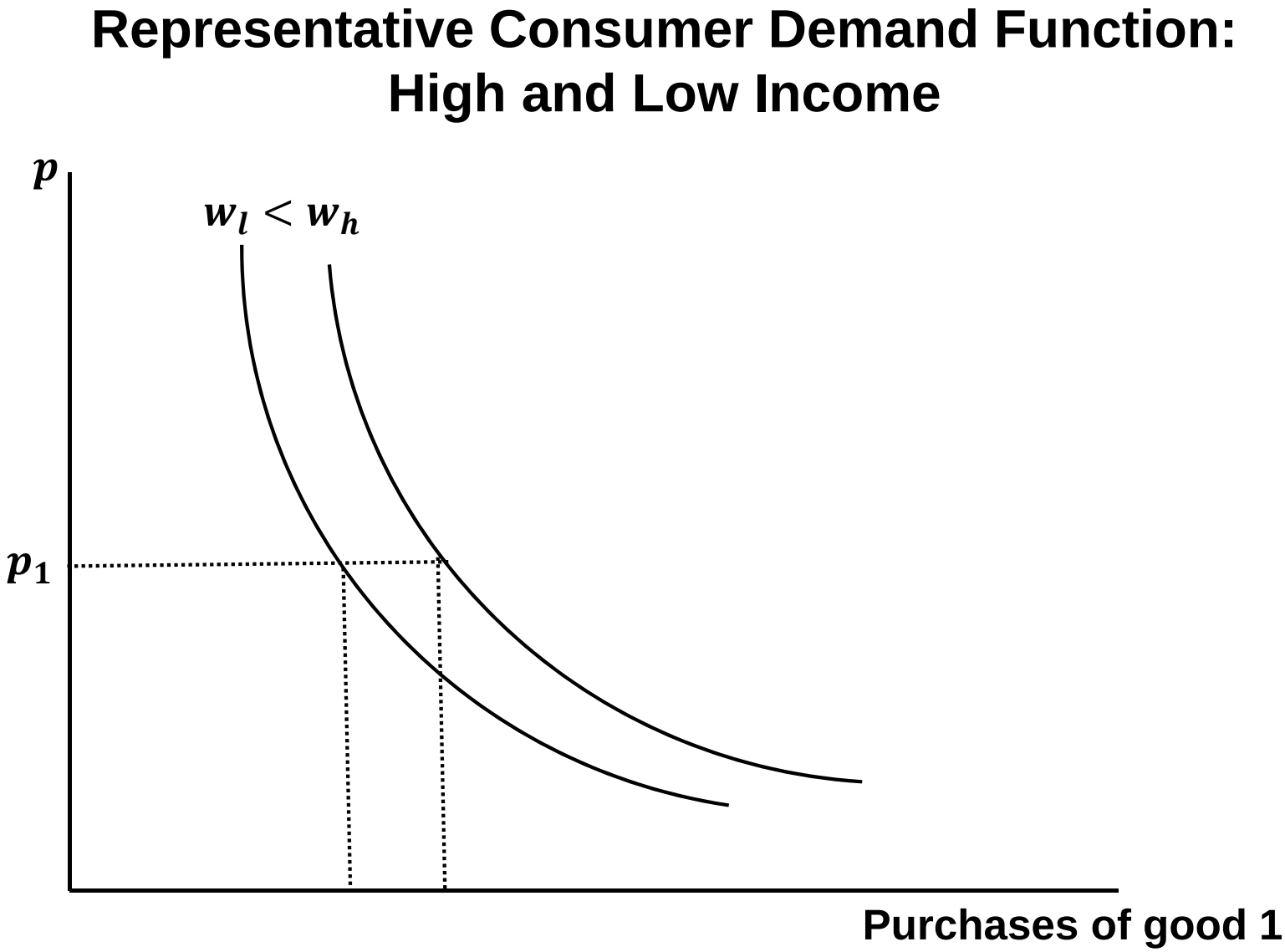
Demand Function: Logarithms



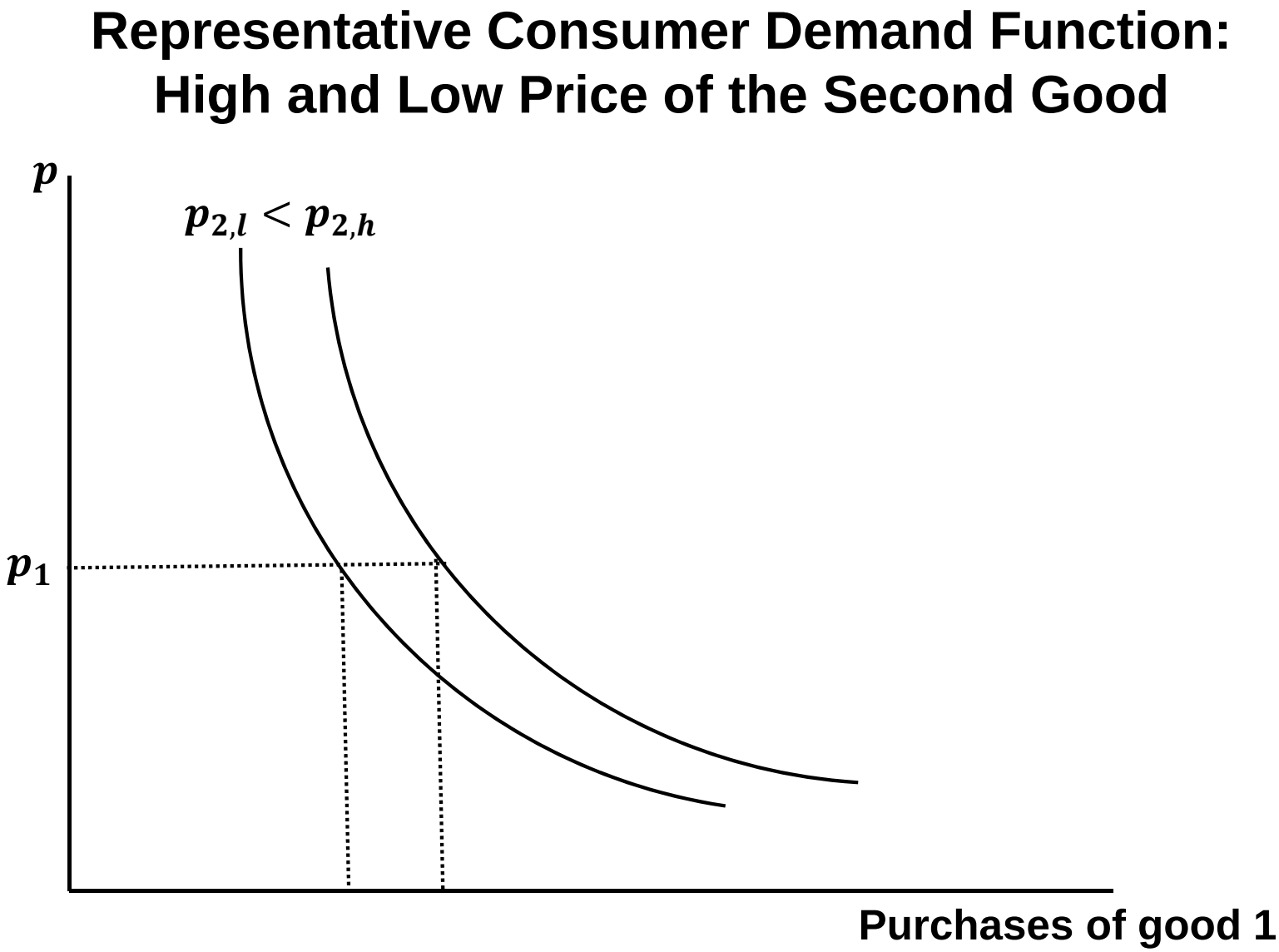
Demand Function: Slope



Demand Function: Changes with Income



Demand Function: Prices of Other Goods



Representative Firm

- We assume firm maximizes profits in a perfectly competitive environment
  - The production function is given by

$$y_{ij} = \frac{A_i}{\tau_{ij}} l_{ij}$$

    where y_{ij} is output of location i for j consumers, $\tau_{ij} \geq 1$ is the shipping cost from i to j, $A_i > 0$ the productivity, and $l_{ij} \geq 0$ the corresponding labor used
  - We assume (naturally) that local shipping costs are zero, $\tau_{ii} = 1$

Firm Optimization

- Because there is perfect competition, the price equals to the marginal cost

$$p_{ij} = w_i \tau_{ij} / A_i$$

  - Prices increase with shipping costs, origin labor cost; decrease with origin productivity
  - What are the relative prices for a given source?

$$\frac{p_{ij}}{p_{ii}} = \frac{\tau_{ij}}{\tau_{ii}} = \tau_{ij}$$

    - We implicitly used this relationship (that preconditions arbitrage) in the previous lectures to derive shipping costs
  - What are the relative prices for a given destination?

$$\frac{p_{i'j}}{p_{ij}} = \frac{w_{i'} \tau_{i'j} / A_{i'}}{w_i \tau_{ij} / A_i} = \frac{A_i}{w_i} \times \frac{w_{i'}}{A_{i'}} \times \frac{\tau_{i'j}}{\tau_{ij}}$$

- Now let us combine consumer and firm choices

Trade Shares in Terms of Wages

- Let us look at location 1 's exports now
- Use firm optimization, $p_{1j} = \tau_{1j} w_1 / A_1$ and $p_{2j} = \tau_{2j} w_2 / A_2$. Substitute in solution for consumer shares of each country's product, equation (9):

$$\lambda_{1j} = \frac{p_{1j}^{1-\sigma}}{p_{1j}^{1-\sigma} + p_{2j}^{1-\sigma}} = \frac{\left(\tau_{1j} w_1 / A_1\right)^{1-\sigma}}{\left(\tau_{1j} w_1 / A_1\right)^{1-\sigma} + \left(\tau_{2j} w_2 / A_2\right)^{1-\sigma}} \tag{11}$$

  - Trade decreases with bilateral trade costs, τ_{1j}
  - Elasticity of substitution, σ, plays a key role in response of sales to costs τ_{ij}, w_i
    - To see this, you can think of the relative shares

$$\frac{\lambda_{1j}}{\lambda_{2j}} = \frac{\left(\tau_{1j} w_1 / A_1\right)^{1-\sigma}}{\left(\tau_{2j} w_2 / A_2\right)^{1-\sigma}} \implies$$

$$\ln\left(\frac{\lambda_{1j}}{\lambda_{2j}}\right) = (1-\sigma)\ln\left(\frac{\tau_{1j}}{\tau_{2j}}\right) + (1-\sigma)\ln\left(\frac{w_1}{A_1}\right) - (1-\sigma)\ln\left(\frac{w_2}{A_2}\right)$$

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