The Economics of Space 433: Lectures 7 and 8

Demand and Supply in Space

Costas Arkolakis[1]

[1]Yale University

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The Economics of Space

- We defined the essential elements for our model
    - Productivities, amenities, spatial links

The Economics of Space

- We defined the essential elements for our model
  - Productivities, amenities, spatial links
- We will now attempt to illustrate the simple economics of demand and supply in space that will guide our analysis in the rest of the course
  - Remarkably the basic spatial model and its extensions can be studied using these basic economic notions
  - This analysis draws ideas from Allen, Arkolakis, Takahashi '20

Setup

- Set of locations $S = \{1, 2, ..., S\}$
  - Each location produces a differentiated commodity with elasticity σ
  - Same notation for locations and goods
    - Convention: origin denoted by i, destination by j
  - Population of location $j \in S$, denoted by L_j
- Total population

$$\bar{L} = \sum_j L_j \tag{1}$$

  - Topography economy: Productivities, amenities, trade costs

Notational Conventions

- Let us talk a little bit about the summation notation. Consider

$$\sum_j L_j = L_1 + L_2 + ... + L_{S-1} + L_S$$

  - Notice that it does not matter which index we use
    - i.e., $\sum_j L_j = \sum_i L_i$: this is just renaming the index

- Now consider $\sum_j \bar{c}_i L_j$, $\bar{c}_i \in \mathbb{R}$.Here, we do need to be mindful of the indices
  - In this case we are summing over j. We can take out a common factor

$$\sum_j \bar{c}_i L_j = \bar{c}_i L_1 + \bar{c}_i L_2 + ... + \bar{c}_i L_{j-1} + \bar{c}_i L_j$$

- Another case is when we have origin and/or destination
  - E.g. X_{ij} and $X_{ij'}$ will denote the sales from i to two different destinations, j, j'

Roadmap

- **Aggregate Demand in a Location**
- Aggregate Supply in a Location
- The Concept of General Equilibrium
- General Equilibrium Characterization: Demand and Supply Together at Last

Bilateral and Total Demand

- The total sales between two locations are $X_{ij} = p_{ij}c_{ij} \times L_j$ where c_{ij} is the individual consumption
  - Recall from lectures 3-4 and the homework (for the many locations case) that

$$\lambda_{ij} = \frac{p_{ij}c_{ij}}{w_j} = \frac{p_{ij}c_{ij}}{\sum_i p_{ij}c_{ij}} = \frac{p_{ij}^{1-\sigma}}{P_j^{1-\sigma}}$$

where by $P_j^{1-\sigma} = \sum_i p_{ij}^{1-\sigma}$

- Thus, the bilateral sales become

$$X_{ij} = p_{ij}c_{ij} \times L_j = \lambda_{ij} \times w_j \times L_j = \frac{p_{ij}^{1-\sigma}}{P_j^{1-\sigma}} w_j L_j$$

Bilateral and Total Demand

- Define p_i as the production cost so that $p_{ij} = p_i \tau_{ij}$ where τ_{ij} is the shipping cost
  - Total demand for location i is given by $Y_i = \sum_j X_{ij}$ so that

$$Y_i = p_i^{1-\sigma} \underbrace{\sum_j \tau_{ij}^{1-\sigma} \frac{w_j L_j}{P_j^{1-\sigma}}}_{\text{producer market access}}$$

where we define **"producer market access"** (see Anderson van Wincoop '03 and Redding Venables '04) as

$$\Pi_i \equiv \sum_j \tau_{ij}^{1-\sigma} \frac{w_j L_j}{P_j^{1-\sigma}}$$

that summarizes selling potential of market i

Aggregate Demand

- Define $Q_i \equiv Y_i / p_i$ the average quantities sold by i. Rearranging

$$\ln p_i = -\frac{1}{\sigma} \ln Q_i + \frac{1}{\sigma} \ln \Pi_i$$

- An explicit equation for demand curve in location i
- It behaves exactly like a standard demand function
  - Downward sloping relationship between p_i, Q_i

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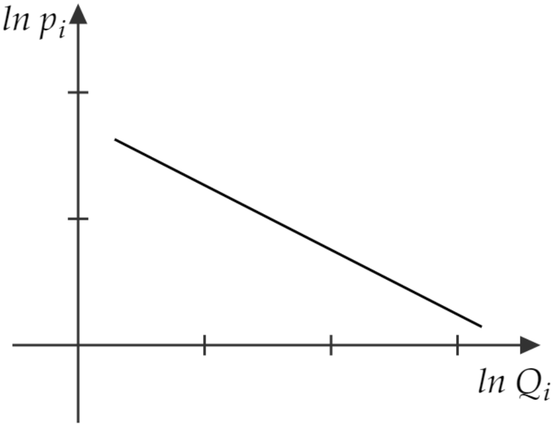
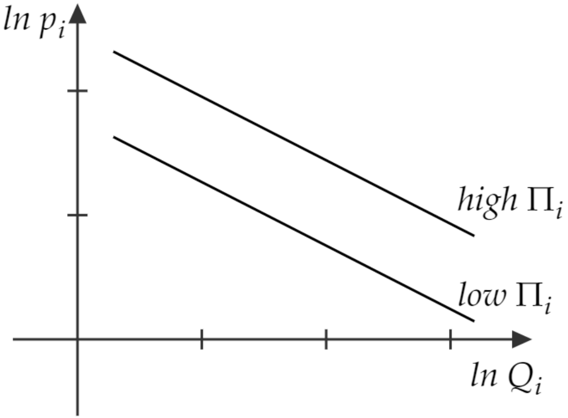
  - Downward sloping relationship between p_i, Q_i
  - Π_i is a demand shifter: higher market potential shift demand curve upwards

Aggregate Demand Shifts



Aggregate Demand Shifts



Aggregate Demand

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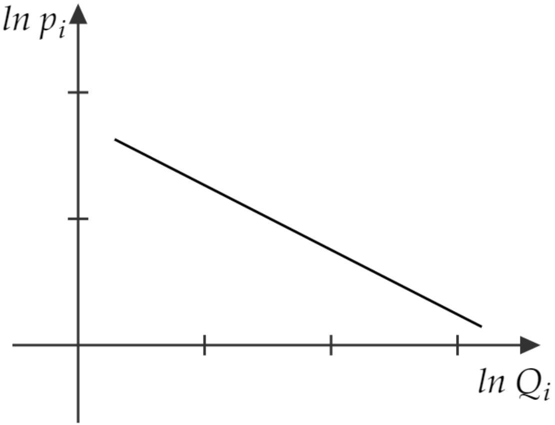
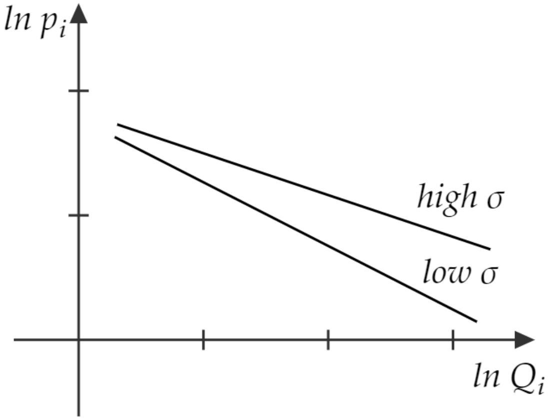
$$\ln p_i = -\frac{1}{\sigma} \ln (Q_i) + \frac{1}{\sigma} \ln \Pi_i$$

- An explicit equation for demand curve in location i
- It behaves exactly like a standard demand function
  - Downward sloping relationship between p_i, Q_i
  - Higher of substitution flattens the demand curve

Aggregate Demand and Elasticity of Substitution



Changes in the Elasticity of Substitutions



No Trade Costs and Homogeneous Products

- If there are no trade costs, $\tau_{ij} = 1$ for all i and j

$$\Pi_i = \sum_j \tau_{ij}^{1-\sigma} \frac{w_j L_j}{P_j^{1-\sigma}} = \sum_j \frac{w_j L_j}{P_j^{1-\sigma}} = \frac{1}{P^{1-\sigma}} \sum_j w_j L_j = \frac{Y}{P^{1-\sigma}}$$

  where we define total income in the economy $Y = \sum_j w_j L_j$

  - The price index is also the same across locations. Normalize $P^{1-\sigma} = 1$

- In this case,

$$\ln p_i = -\frac{1}{\sigma} \ln \left(Q_i \right) + \frac{1}{\sigma} \ln Y$$

  and the shifter is simply the total world (real) income

- Finally note that $\sigma \to \infty$ and $\tau_{ij} = 1$ takes us back to the Rosen Roback model

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  - Demand curve is flat: fixed price in each location

Aggregate Demand for Labor

- Using the same logic, we can derive the aggregate demand for labor
- Note that $p_i = w_i / A_i$. Therefore, in i

$$\ln p_i = \ln w_i - \ln A_i \implies$$

$$\ln w_i = -\frac{1}{\sigma} \ln (Q_i) + \ln A_i + \frac{1}{\sigma} \ln \Pi_i$$

- Now notice that productivity acts as an additional shifter for labor demand
  - Higher producer market access AND higher productivity means higher demand for labor

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- Aggregate Demand in a Location
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Supply of Labor

- Recall that from welfare equalization

$$W = C_j u_j \iff W = C_j \bar{u}_j L_j^{-\beta}$$

$$L_j = \frac{\left(C_j \bar{u}_j\right)^{1/\beta}}{W^{1/\beta}}$$

- It can be proven that with the demand we have used, we always have that $P_j C_j = w_j$

$$L_j = \frac{\left(w_j \bar{u}_j / P_j\right)^{1/\beta}}{W^{1/\beta}} \tag{2}$$

  - Where $P_j \equiv \left(\sum_i p_{ij}^{1-\sigma}\right)^{1/(1-\sigma)}$ summarizes total access of consumer in market j from all locations i.e. **"consumer market access"**

Aggregate Supply of Labor in a Location

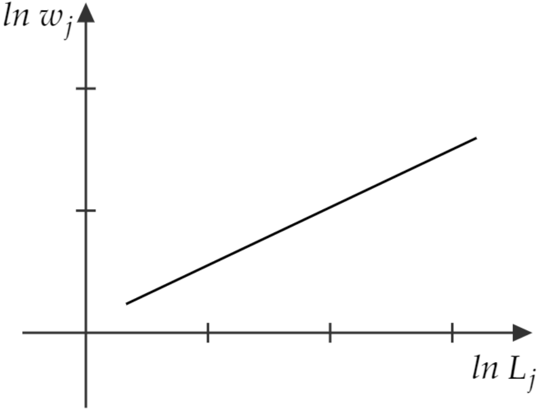
- Taking logs

$$\ln L_j = \frac{1}{\beta} \ln w_j + \frac{1}{\beta} \ln \frac{\bar{u}_j}{P_j W} \iff$$

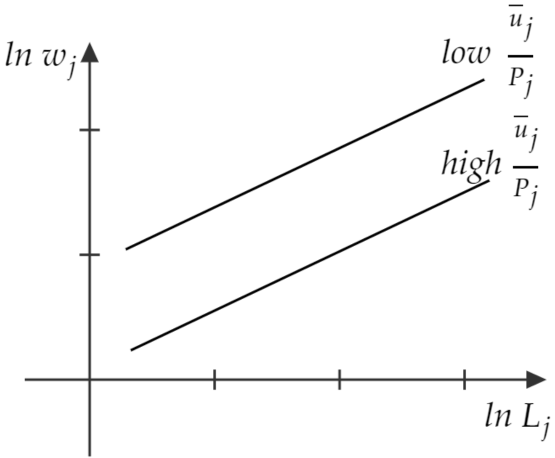
$$\ln w_j = \beta \ln L_j - \ln \frac{\bar{u}_j}{P_j} + \ln W \tag{3}$$

- Notice that now the shifter of aggregate labor supply is the ratio of exogenous amenity and the price index of a location
  - Higher amenity AND lower price index shift aggregate supply of labor downwards

Aggregate Supply Shifts



Aggregate Supply Shifts



The Elasticity of Labor Supply

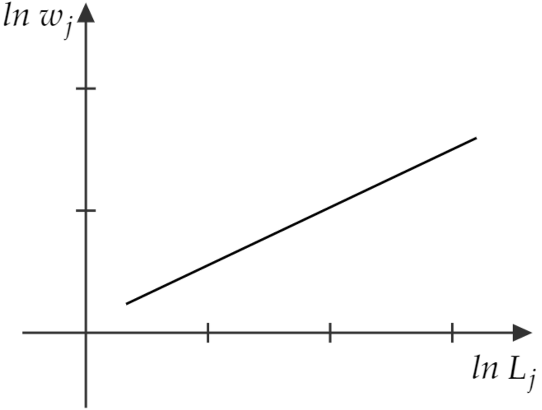
- Taking logs

$$\ln L_j = \frac{1}{\beta} \ln w_j + \frac{1}{\beta} \ln \frac{\bar{u}_j}{P_j} - \frac{1}{\beta} \ln W \iff$$

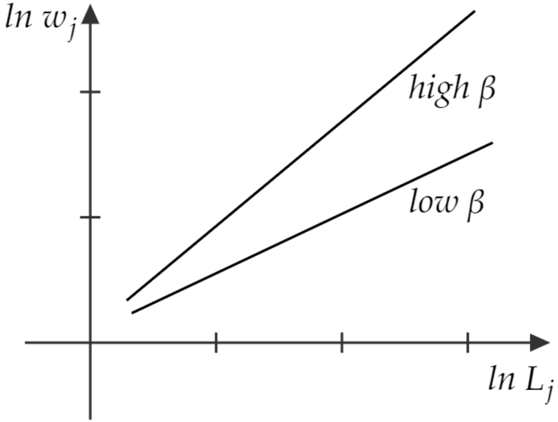
$$\ln w_j = \beta \ln L_j - \ln \frac{\bar{u}_j}{P_j} + \ln W$$

  - The ratio $1/\beta$ here represents the elasticity of labor supply to (real) wages
  - $\beta \to \infty$ means a vertical labor supply function. Labor does not respond to wages

Aggregate Supply And Labor Supply Elasticity



Changes in the Labor Supply Elasticity



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- **The Concept of General Equilibrium**
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The Concept of General Equilibrium

- So far we have taken wages and labor as given
- To solve them, we need to solve for the joint interactions of firms and consumers
- Therefore, we need to combine aggregate supply and aggregate demand
  - This concept of jointly solving the decision of firms and consumers interacting via wages is called ''general equilibrium''

Aggregate Demand for Labor

- We consider general equilibrium,
  - We need to write aggregate demand purely in terms of labor demand

- Notice that in our model, $Q_i = L_i \times A_i$. Using the equation for labor demand

$$\ln w_i = -\frac{1}{\sigma} \ln (L_i \times A_i) + \ln A_i + \frac{1}{\sigma} \ln \Pi_i \iff$$

$$\ln w_i = -\frac{1}{\sigma} \ln L_i + \frac{\sigma - 1}{\sigma} \ln A_i + \frac{1}{\sigma} \ln \Pi_i$$

Aggregate Demand for Labor: Labor Demand Slope

- Furthermore, as $A_i = \bar{A}_i L_i^{\alpha}$,

$$\ln w_i = -\frac{1}{\sigma} \ln L_i + \frac{\sigma - 1}{\sigma} \ln \left(\bar{A}_i L_i^{\alpha} \right) + \frac{1}{\sigma} \ln \Pi_i \iff$$

$$\ln w_i = \frac{(\sigma - 1)\alpha - 1}{\sigma} \ln L_i + \frac{\sigma - 1}{\sigma} \ln \bar{A}_i + \frac{1}{\sigma} \ln \Pi_i$$

  - Notice: because of agglomeration feedback, α cannot get too large
  - If $\alpha(\sigma - 1) > 1$, labor demand is upward sloping...(!)

Aggregate Demand and Market Access

- We note (I ll give it away for now, as it requires some algebra) that if $\tau_{ij} = \tau_{ji}$, i.e. trade costs are symmetric,

$$\Pi_i^{\frac{1}{1-\sigma}} = P_i$$

  - If $\tau_{ij} = \tau_{ji}$ decreases (e.g. a new bridge was build, lower tarrifs) with $\sigma > 1$
    - P_i decreases (cheaper goods for consumer in i)
    - Π_i increases (higher demand for goods in i)
- Therefore

$$\ln w_i = \frac{(\sigma - 1)\alpha - 1}{\sigma} \ln L_i + \frac{\sigma - 1}{\sigma} \ln \bar{A}_i + \frac{1 - \sigma}{\sigma} \ln P_i \tag{4}$$

  - Now, we can combine supply and labor demand

Aggregate Demand and Supply: Together at Last

- We now combine aggregate labor demand, equation 4, and aggregate labor supply, equation 3.
  - In particular, by equating them we obtain

$$\beta \ln L_i - \ln \frac{\bar{u}_i}{P_i} + \ln W = \frac{(\sigma-1)\,\alpha - 1}{\sigma} \ln L_i + \frac{\sigma - 1}{\sigma} \ln \bar{A}_i + \frac{1-\sigma}{\sigma} \ln P_i \iff$$

$$\frac{\beta\sigma - (\sigma-1)\,\alpha + 1}{\sigma} \ln L_i = \frac{\sigma-1}{\sigma} \ln \bar{A}_i + \ln \bar{u}_i + \frac{1-2\sigma}{\sigma} \ln P_i - \ln W \iff$$

$$(\beta\sigma - (\sigma-1)\,\alpha + 1) \ln L_i = (\sigma-1) \ln \bar{A}_i + \sigma \ln \bar{u}_i + (1-2\sigma) \ln P_i - \sigma \ln W$$

- This is the solution for labor as a function of geography and market access
  - In fact, unless $\underbrace{\beta\sigma}_{\text{supply slope}} > \underbrace{(\sigma-1)\,\alpha - 1}_{\text{demand slope}}$, the equilibrium is not "well behaved"
  - In fact, this is a necessary, not a sufficient condition

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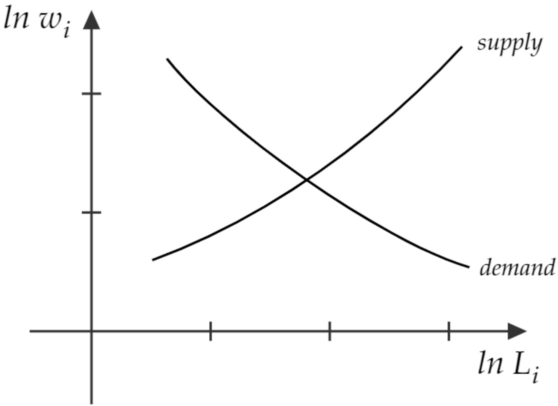
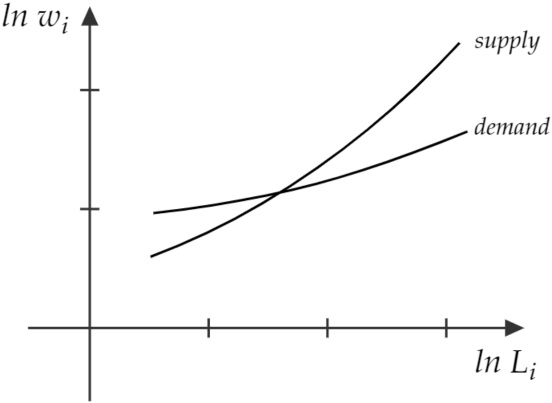
Aggregate Demand and Supply: Together at Last

- We finally want to analyze the equilibrium
  - Demand and Supply come together
- Equilibrium solution in the well behaved case
  - Study the effects of changes in exogeneous amenities, productivities, and the consumer market access P_i
  - This is an insightful but not final analysis, as $P_i\left(\{w_j, L_j\}_j\right)$
  - We will return with a more rigorous analysis later in the course

General Equilibrium: Demand & Supply in Space

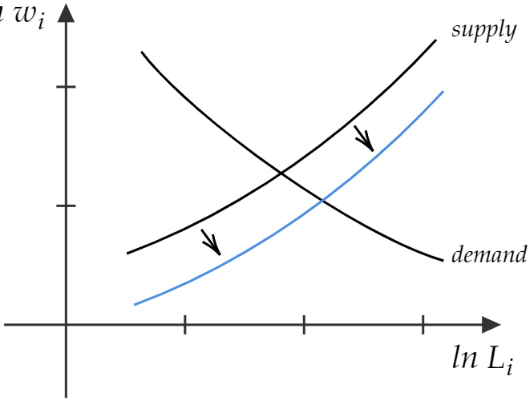


General Equilibrium: Demand & Supply in Space



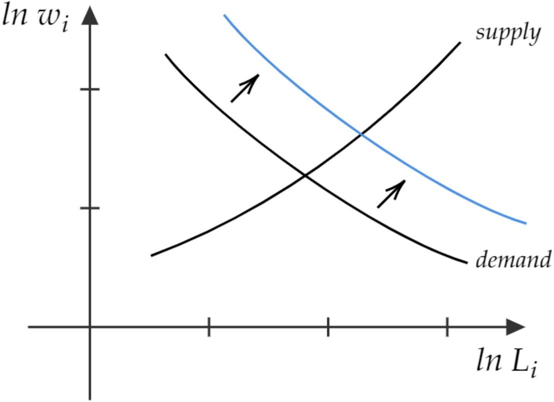
Labor demand and supply when demand is upward sloping

General Equilibrium: Changes in Amenities



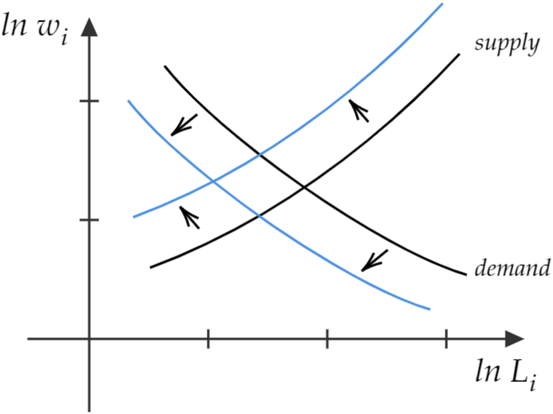
Labor demand and supply where $\bar{u}_i$ increases

General Equilibrium: Changes in Productivities



Labor demand and supply where $\bar{A}_i$ increases

General Equilibrium: Changes in Market Access



Labor demand and supply where P_i increases

How Does Space Matter for the Allocation of Economic Activity

- Recall access terms are simply absent in a Rosen-Roback framework
  - With no trade frictions $P_i = P$ and $\Pi_i = \Pi = \left(\sum_j w_j L_j\right) / P^{1-\sigma}$
  - Other than that, the equations for wages and labor are the same

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  - Other than that, the equations for wages and labor are the same

- Consumer and producer market access, therefore, shape the allocation of wages and labor across space
  - They summarize what space is all about.
  - Of course P_i, Π_i are themselves a function of (all) wages and labor and we will see how to solve for all these in the next classes
  - But coming up next, we take space seriously!

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