The Economics of Space 433: Section 2

Monocentric model and Rosen-Roback model

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Plan

- Review Rosen-Roback model
- Review Monocentric model
  - Finding an equilibrium agriculture price at the center

Rosen-Roback Model: Set up

- Space:
  - An arbitrary number of locations
  - No barriers of moving goods or people across space
- Producers
  - A homogeneous good is produced at all location
  - Perfect competition
  - CRS (linear) technology using labor with productivity A_i
  - **Agglomeration on labor productivity** $A_i = \bar{A}_i L_i^{\alpha}, \alpha \geq 0$
- Homogeneous consumers
  - Welfare $W_i = C_i u_i$ where C_i = consumption, u_i = amenity
  - Budget constraint: $p_i C_i = w_i$
  - **(Negative) Externalities on amenity** $u_i = \bar{u}_i L_i^{-\beta}, \beta > \alpha$
- No trade costs, homogeneous goods

Rosen-Roback Model: Equilibrium

- Producer problem: $A_i p_i = w_i$
- Consumer problem: $W_i = \frac{w_i}{p_i} u_i$ (i.e., $C_i = \frac{w_i}{p_i}$)
- No arbitrage: $p_i = p$. Let $p = 1$ (without loss)
- Putting all together

$$W_i = \bar{A}_i \bar{u}_i L_i^{\alpha-\beta}$$

Rosen-Roback Model: Equilibrium

- We need to find (L_i, W_i) that satisfies welfare equalization (free mobility) and L_i adding up to the total population

$$W_i = W \quad \forall i \tag{1}$$

$$\sum_i L_i = \bar{L} \tag{2}$$

  - Note: (1) is for all i with $L_i > 0$
  - $L_i = 0$ for some i can be an equilibrium? (recall $\beta > \alpha$)

Rosen-Roback Model: Equilibrium

- Begin with (1) welfare equalization condition $W = W_i = \bar{A}_i \bar{u}_i L_i^{\alpha-\beta}$:

$$L_i = \left(\frac{W}{\bar{A}_i \bar{u}_i} \right)^{\frac{1}{\alpha-\beta}} = W^{\frac{1}{\alpha-\beta}} (\bar{A}_i \bar{u}_i)^{\frac{1}{\beta-\alpha}} \quad (3)$$

  - $W = W_i$ is still unknown, which will be determined in the equilibrium
  - But, we still have on more condition (2) that can pin down W

Rosen-Roback Model: Equilibrium

- Plug in (3) to (2),

$$\bar{L} = \sum L_i = \sum_i \left(\frac{W}{\bar{A}_i \bar{u}_i} \right)^{\frac{1}{\alpha - \beta}}$$

$$\Rightarrow \quad W = \left(\frac{\sum_i (\bar{A}_i \bar{u}_i)^{\frac{1}{\beta - \alpha}}}{\bar{L}} \right)^{\beta - \alpha}$$

  - Properties of W: (1) Increasing in $\bar{A}_i$ and $\bar{u}_i$, (2) Decreasing in $\bar{L}$

- Plugging this W into (3) L_i, we have an equilibrium

$$L_i = \left(\frac{W}{\bar{A}_i \bar{u}_i} \right)^{\frac{1}{\alpha - \beta}} = \frac{(\bar{A}_i \bar{u}_i)^{\frac{1}{\beta - \alpha}}}{\sum_j (\bar{A}_j \bar{u}_j)^{\frac{1}{\beta - \alpha}}} \bar{L}$$

- We solved an equilibrium (L_i, W_i)!

Rosen-Roback Model: Equilibrium

- **Another strategy**: observe that (3) determines the share of population across i

$$L_i = \left(\frac{W}{\bar{A}_i \bar{u}_i}\right)^{\frac{1}{\alpha-\beta}} \Rightarrow \frac{L_i}{L_j} = \left(\frac{\bar{A}_j \bar{u}_j}{\bar{A}_i \bar{u}_i}\right)^{\frac{1}{\alpha-\beta}}$$

$$\Rightarrow \bar{L} = \sum_i L_i = \sum_i \left(\frac{\bar{A}_j \bar{u}_j}{\bar{A}_i \bar{u}_i}\right)^{\frac{1}{\alpha-\beta}} L_j$$

$$\Rightarrow L_j = \frac{(\bar{A}_j \bar{u}_j)^{\frac{1}{\beta-\alpha}}}{\sum_k (\bar{A}_k \bar{u}_k)^{\frac{1}{\beta-\alpha}}} \bar{L}$$

Rosen-Roback Model: Equilibrium properties

- Observe that L_i depends on...

$$L_i = \frac{(\bar{A}_i \bar{u}_i)^{\frac{1}{\beta-\alpha}}}{\sum_j (\bar{A}_j \bar{u}_j)^{\frac{1}{\beta-\alpha}}} \bar{L}$$

  - i's *relative* attractiveness: What happens if $\bar{A}_i \to a\bar{A}_i$ for all i?
  - $\beta - \alpha$: What happens if $\beta - \alpha$ is large? (e.g. $\beta - \alpha \to \infty$)
  - $\bar{L}$: What happens if the population doubles?

Monocentric City

- Sectors: manufacturing M and agriculture A
- Space:
  - An arbitrary number of locations is allowed. Only a subset S is populated
  - $c \in S$ is the city center. $f \in S$ is the agriculture frontier location
  - $\tau_{j,s}$ = iceberg trade cost from c to j of good s which is continuous and monotonically increasing in the distance $d(c, j)$
  - 1 measure of land for each i

Monocentric City: Model

- M producer in c: Unit output per worker

- A landlord in i ($\neq c$):
  - Cannot move and consume locally
  - Has one unit of land
  - Inputs $\frac{1}{\bar{A}_{i,A}}$ of labor and one unit of land to produce a unit of A
  - Trade only with c

- Homogeneous consumers with C-D preference spending μ share on M
  - Total population $\bar{L}$
  - Free mobility across i

Monocentric City: Equilibrium

- **Consumers:** solving a utility maximization problem (last week TF session)
  - C-D utility price index $P_i = p_{i,M}^{\mu} p_{i,A}^{1-\mu}$
  - Welfare $W_{i,s} = \frac{w_{i,s}}{P_i} = \frac{w_{i,s}}{p_{i,M}^{\mu} p_{i,A}^{1-\mu}}$
- M **producer in** c: $p_{c,M} = w_{c,M}$
- A **producer (landlords) in** i:

$$\max \pi_{i,A} = \left\{ \underbrace{0}_{\text{inactive}}, \underbrace{p_{i,A} - \frac{w_{i,A}}{\bar{A}_{i,A}}}_{\text{active}} \right\}$$

  - Will produce if $p_{i,A} = \frac{p_{c,A}}{\tau_{i,A}} \geq \frac{w_{i,A}}{\bar{A}_{i,A}}$
  - Focus on the parameters that $S = [0, f]$ (recall the condition in class slides!)
  - We only need to solve for the cutoff value f (not complicated S!)
- No arbitrage: $p_{j,M} = p_{c,M}\tau_{j,M}$, $p_{c,A} = p_{i,A}\tau_{i,A}$
  - wlog, assume $p_{c,M} = 1$

Monocentric City: Equilibrium

- We need to find $(p_{i,s}, W_{i,s}, L_{i,s}, w_{i,s}, f)$ that satisfies welfare equalization and agriculture market clearing:

$$W_{i,A} = W_{c,M} \quad \forall i \in S \tag{4}$$

$$(1-\mu)Y_c = \int_{i \in S} X_{ic,A} di \tag{5}$$

  where Y_c = total income of c from selling M and $X_{ic,A}$ = the import value of A from i to c

  - Welfare equalization is the same as Rosen-Roback.
    We use location c as a reference location. Why?
  - Condition (5) means the total spending on A from c (recall that $i \neq c$ exports to only c) should be equal to the total sales from all i to c

Monocentric City

- From the class, after some derivations, we need to solve for $(p_{c,A}, f)$. Start with condition (4) (again from the class)

$$p_{c,A} = \frac{(\tau_{f,M})^{\mu}(\tau_{f,A})^{\mu}}{\bar{A}_{f,A}} \tag{6}$$

- For condition (5),
  - $Y_c = w_{c,M} L_{c,M} = \bar{L} - \int_{i \in S} \frac{1}{\bar{A}_{i,A}} di$ (recall $w_{c,M} = p_{c,M} = 1$)
  - $X_{i,c} = \mu p_{i,A}$ (spending share on M $\times$ total income of i from A)
  - Putting these together

$$(1-\mu)\left(\bar{L} - \int_0^f \frac{1}{\bar{A}_{i,A}} di\right) = \mu \int_0^f \frac{p_{c,A}}{\tau_{i,A}} di \tag{7}$$

- Conditions (6) and (7) give $(p_{c,A}, f)$

Monocentric City

Recall (6) $p_{c,A} = \frac{(\tau_{f,M})^{\mu}(\tau_{f,A})^{\mu}}{\bar{A}_{f,A}}$ and (7)

$$(1-\mu)\left(\bar{L} - \int_0^f \frac{1}{\bar{A}_{i,A}} di\right) = \mu \int_0^f \frac{p_{c,A}}{\tau_{i,A}} di$$

Example:

- Suppose $\bar{A}_{i,A} = 1$ for all i, $\tau_{i,s} = e^{\tau_s d(c,i)}$ and $d(c,i) = i$
- Try to find an equilibrium:

$$\begin{aligned}(1-\mu)(\bar{L} - f) &= \mu e^{\mu\tau_M f + \mu\tau_A f} \int_0^f e^{-\tau_A i} di \\ &= \mu e^{\mu\tau_M f + \mu\tau_A f} \left[-\frac{1}{\tau_A} e^{-\tau_A i} \right]_0^f \\ &= \mu e^{\mu(\tau_M + \tau_A) f} \frac{1}{\tau_A}(1 - e^{-\tau_A f})\end{aligned}$$